THE

PLAIN CALCULATOR;

BEING AN

ELEMENTARY ARITHMETIC,

BASED ON THE

INHERENT PROPERTIES OF NUMBERS.

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Increase—Equality—Decrease.

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TO

THE PEOPLE OF THE UNITED STATES,

TO WHOM IS GIVEN THE HIGH PRIVILEGE

OF PROMOTING THE GREATEST GOOD OF THE GREATEST NUMBER;

AND WHO HOLD

THAT MOST ILLUSTRIOUS WHICH IS MOST USEFUL,

This Volume

IS INSCRIBED BY THEIR HUMBLE FRIEND,

THE AUTHOR.

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PREFACE.

In the first edition of *any* work, the author stands in special need of the indulgence of those whom he addresses; and the circumstances under which this volume appears, require additional allowance from its readers.

The Author arrived in this country more than twelve years ago, intending to examine for himself the state of education in this celebrated nation, whose fame for advancement in every department of true civilization had passed through all Europe.

His favourite branch of knowledge is *Mathematics;* and he was astonished to find that while all the sister sciences and arts were cherished and improved, and all the nobler themes of human destiny and duty were nourished to a giant growth—this single one had been strangely left in the dwarfed, wild state of the dark ages—and that too in its more important and radical parts.

Mathematical truth, like all real wisdom, comes to us from the pure Source of all excellence; its laws are simple, but inexorable; their true end is doubtless to promote peace and good will among men, but this consummation may be expected only in proportion as they are made perspicuous.

Thus impressed, the author determined to attempt improvement in American Arithmetic. That he saw reason for change may be inferred from the fact that at this time he had no knowledge of the English language—and had reached a time of life when reviewing and preserving, rather than acquiring, is most congenial.

Passing into the rural districts, where German and English were alternately spoken, he sought to attain a knowledge of the latter in its colloquial form, and then impart his views in familiar lectures. He expected to encounter difficulties; but not such as have crossed his path. The stubborn peculiarities of the language he anticipated to some extent, and after several years' application he was able to instruct classes satisfactorily in Vermont, Connecticut, New York, New Jersey, Pennsylvania, Maryland, Virginia, and South Carolina. He had proceeded as far South as Charleston, when the great disturbance of business occurred in 1837, which so unsettled the public mind as to terminate abruptly his immediate prospects, and hinder a return of such ever since.

About a year ago the present work suggested itself; and its pursuit has been maintained through difficulties that cannot be estimated, and need not be named. Hopelessly inexpert in the language, and unused to critical correction of the Press, much imperfection in detail will be found. The principles alone are offered fearlessly, and will prove worth a careful examination.

—

The *Original* points of the work may be thus stated :

The *Properties of Figures and Numbers* have been greatly extended—and a table of *Prime Factors* is given as far as 10,000.

Cancellation and *Contraction* has been applied to all the usual transactions of business: having adopted as a maxim that in proportion as we can dispense with figures in calculation, we diminish the means of error in our conclusions.

Reduction has been simplified in its operations. The term has been retained as proper in its antiquated sense (to bring *back*.) It is thus applied in surgery.

The treatment of *Fractions* has been improved by his simplified means of finding the greatest common factor and the least common multiple.

The Rule of Solution in *Alligation* will be found to differ from any hitherto offered in the United States.

The difference between *Discount* and *Interest* is stated unequivocally, as applied to ordinary transactions, and also in Equation of Payments.

Finally; the theory of *Direct* and *Inverse Proportion* is specially commended to all arithmeticians.

The unmeaning distinctions formerly adopted, viz. "*more* requiring *more*," and "*more* requiring *less*," and *vice versa*, have already been discarded in some publications; but merely avoiding the imperfect definitions of predecessors,

will not render those principles intelligible which have been misunderstood.

The criterion here proposed addresses both our judgment and perception—it requires us to examine the circumstances as well as the conditions of a question.

In *Simple* or *Single Proportion* we have an original quantity and its equivalent, whereby to find an equivalent for a second given quantity. In such cases no inverse or indirect circumstances can arise—according to the axiom that like causes under similar circumstances produce like effects.

In *Compound Proportion* we have three or more quantities given, which sustain a certain relation to each other, whereby to find a new quantity, such as shall hold a similar relation with certain other given quantities. We distinguish said quantities into *active* and *passive*, or into *cause* and *effect*. Time and tools produce labour, men and days create or consume value; length, breadth and depth, are the *cause* of solid contents.

Direct Proportion is where the unknown quantity or answer is a *passive* term, or product.

Inverse Proportion exists where the requisition is for an *active* term, or producer.

We have an invariable rule of statement, which places Divisors on the left side and Multipliers on the right, and thus sets the points in controversy in the most desirable

position for examination. If the question be *direct*, according to the above test, we proceed at once to find the answer by multiplication and division; but if it be *inverse*, we "invert" (or properly *transpose*) the *given active terms*, (except that of similar name with the answer,) writing them on the side of the perpendicular line opposite to that where our statement placed them,—and then proceed to multiply and divide as in all other solutions.

The definition hitherto applied to Indirect Proportion seems to have been adopted by our predecessors from necessity. In order to attain the true result they had to *invert* certain of the terms: with this fact they were satisfied—*why* this was so never disturbed their faith. In our principles will be found not only the law, but the *reasons* for its *Direct Proportion* gives us a full cause and an effect, and another full cause to which we are to find an appropriate effect; but *Inverse Proportion* provides a cause and an effect, and another effect and *part* of a cause, by which we are to find the *remainder* of said cause.

These few pages are now offered to the Public with equal candour and humility. Holding the doctrine that the possessors of valuable knowledge are bound to impart the same for general benefit, the author makes no pretensions to exalted philanthropy. If the present elementary volume be well received, a second may be prepared, in which these principles will be applied to the higher branches of Arith-

metic. The CALCULATOR is intended to be a sort of every-man's every-day Book; and the examples are all of a strictly practical description. It is to be regretted that so many *purely curious* questions in calculation are exhibited to children, before a proper knowledge of the fundamental principles of Arithmetic have been acquired. The attractive character of these phantoms lead away the attention from the sober realities of life; and when at length the mechanic or merchant enters real business, he finds that although the morning has been wasted "by authority," the fervour of noon makes no allowance therefor, and night hastens as though he were prepared for it.

That this little treatise will accomplish all the good within his intentions, is not likely; but that it will be useful to the rising generation of the United States, is the sincere belief of

THE AUTHOR.

THE

PLAIN CALCULATOR.

ARITHMETIC is the Science of Calculation, and teaches us how to ascertain and express *Quantities*.

Quantities are either perfect or imperfect.

Perfect quantities are composed of complete amounts, called *whole numbers*.

Imperfect quantities are incomplete amounts, and are called *fractions*.

The means by which calculation is performed have been classed under the following general rules. Notation, Numeration, Addition, Subtraction, Multiplication and Division.

ARITHMETICAL NOTATION

Teaches how to express quantities to the *eye*, and relates to the *shape* of the characters used. Perfect quantities are thus written,

1	2	3	4	5	6	7	8	9	0
One	Two	Three	Four	Five	Six	Seven	Eight	Nine	Cipher.

The first *nine* of these characters have a *positive* individual force or value—the tenth (0) has only a *relative* meaning; as, while 4 means *four*, and 7 means *seven*, &c. 0 or 000 means nothing; but in connexion with *positive* characters, the cipher has great importance, as 1000 expresses one *thousand*.

Imperfect quantities are called Fractions; they are expressed by two numbers placed one above the other, with a horizontal line between, thus, $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{3}$ $\frac{2}{5}$ $\frac{5}{8}$ &c.

Besides the above there are characters used to show the *relations* between quantities—they serve the same purpose in arithmetic that *prepositions* do in grammar. The following are those most frequently used.

SIGNS.	SIGNIFICAT
$=$	equal, as 100 cents=1
$+$	more, or added to, as 4
$-$	less, or diminished by,
$\times$	multiplied by, as 7 × 6
$\div$	divided by, as 15 ÷ 5=
: :: :	as, is to, so-is, to; as 2
	as 2 is to 5 so is 6 t

NUMERATIO

Teaches the value of figures acc a horizontal series.

Our arithmetical characters (so press all ordinary quantities) ha n: 1st, a positive or arbitrary me rm when standing singly: and 2d, a gnification when placed in compan sociated condition, that our present how to enumerate and express r instance, in the number 6467 t 7 *units*, but in the number 6476 th d in the number 6746 its value i is is shown more fully by the fol

Tens of Millions	Millions	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units	
							9	nine.
						8	9	eighty-nine.
					7	8	9	seven hund
				6	7	8	9	6 thousa
			5	6	7	8	9	56 thousan
		4	5	6	7	8	9	456 thousan
	3	4	5	6	7	8	9	3 million
2	3	4	5	6	7	8	9	23 million
2	3	4	5	6	7	8	9	123 million

...rchant of New York has to pay in
...$783 at 10 per cent. premium, he
...ay the same to a broker to obtain a
...suppose (as was the case lately) that
...l per cent. better than those of Phila-
...iladelphia merchant had received the
...m England, how much must he pay

Solution.

		~~783~~ 87
	~~9~~	~~40~~ 2
premium	$ ~~100~~	~~110~~ 22
Phila.	100	106
	100	4057 68 Ans.
of New York pays		3828
		229 68 diff.

...e the conclusion that the merchant of ...he same article for the same price and ...ansaction $229 68 more than the mer-..., which illustrates plainly the influence ...merce of different states in the union, ...s of the world in general.

...is method of statement, if a question ...l additional conditions, that we are not ...systems, to make separate statements ...set all the conditions as fixed, under-...s usual.

...742lbs. of wool on the following condi-...er cent. tret (*tret* means an allowance ...odities) and to pay for 1lb. net weight ...lvania currency, at 6 per cent. discount; ...gain to B on the same condition, with a ...nt. how much must B pay to A in federal

...on, in *New York Currency*, is taken out of my

SIGNS.	SIGNIFICATIONS.
=	equal, as 100 cents=1 dollar.
+	more, or added to, as 4+3=7.
—	less, or diminished by, as 5—2=3.
×	multiplied by, as 7×6=42.
÷	divided by, as 15÷5=3.
: :: :	as, is to, so-is, to; as 2 : 5 :: 6 : 15, that is as 2 is to 5 so is 6 to 15.

NUMERATION

Teaches the value of figures according to their places in a horizontal series.

Our arithmetical characters (so admirably adapted to express all ordinary quantities) have a two-fold signification: 1st, a positive or arbitrary meaning, denoted by their form when standing singly: and 2d, a relative or combined signification when placed in company; it is to this last or associated condition, that our present rule applies, directing us how to enumerate and express their force and value. For instance, in the number 6467 the figure 7 has a value of 7 *units*, but in the number 6476 the value of 7 is seven *tens*, and in the number 6746 its value is seven *hundreds*, &c. this is shown more fully by the following table.

Hundreds of Millions	Tens of Millions	Millions	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units	
								9	nine.
							8	9	eighty-nine.
						7	8	9	seven hundred and eighty-nine.
					6	7	8	9	6 thousand 789.
				5	6	7	8	9	56 thousand 789.
			4	5	6	7	8	9	456 thousand 789.
		3	4	5	6	7	8	9	3 million 456 thousand 789.
	2	3	4	5	6	7	8	9	23 million 456 thousand 789.
1	2	3	4	5	6	7	8	9	123 million 456 thousand 789.

This table shows that three figures in a period are expressed units, tens, hundreds; therefore the three figures on the right hand express seven hundred and eighty-nine *units;* the next three to the left, four hundred fifty-six *thousand*; again, the *next* three to the left, one hundred twenty-three *millions:* thus the whole will read, "one hundred and twenty-three *millions*, four hundred and fifty-six *thousands*, seven hundred and eighty-nine *units*"—this will include the largest usual business transactions.

ADDITION AND SUBTRACTION.

Addition teaches us how to *unite* distinct quantities.

Subtraction instructs us to *separate* quantities by taking a part from the whole.

RULES.

Addition. Place the numbers to be added accurately underneath each other, units under units, tens under tens, and hundreds under hundreds; commence at the right hand column and add up the units, and carry to the next column the tens contained in its sum; proceed thus for addition till the whole is finished.

Subtraction. Subtract the smaller number from the greater beginning at the unit place; but if the figure above is smaller than the lower, add ten to the upper figure and pay the amount thus borrowed in the next place to the left.

(1) Add		(2) Add		(3) Add
3247		14934		143716*
1498	8216	31493	83576	371419
3471		37149		143714
5734		14734		171349
3714	16822	34718	96600	371493
7374		47148		471348
25038	=25038	180176	=180176	1673039 sum.
	Proof.		Proof.	1529323 * Less top line.
				143716 Proof.

Proofs are made variously; an expert calculator will add upwards and downwards to satisfy himself of correctness; others will, as shown above, divide the sum given into two parts. Accountants, who are often interrupted in their operations, will take a slip of paper and place the sum of each row from the right to the left, and from the left to the right, and compare the result to test its correctness.

(4) Add	243	
	5021	
	7628	
	927	
	64	
	5823	
	742	
29	796	54
46	5009	35
35	325	46
54	7426	29
34004	34004	34004

The numbers here on the right and left hand, represent the addition of each row from the left and right hand, these concurring in the addition of the rows separately, as also in the total, the proof is regarded sufficiently established. This method is one of the best proofs of addition, and if an error has been made it shows in which row it has occurred.

Farther observe, as a fundamental principle, that an equal addition destroys an equal subtraction, and vice versa, wherefore addition can be proved by subtraction, as shown by No. 3, and subtraction can be proved by addition.

(5) Add	271684	(6) 756483	(7) 8156456
	728316	243496	1846545
	643868	656488	2663726
	356132	343573	8136274
	786418	789664	1628572
	648690	689678	
sum	3435108		

We may make the different proofs as above directed for practical exercise.

(1)	From	9438149
	Take	1350348
Remainder or diff.		8087801
Proof.		9438149

Here we may comprehend at once that if we subtract 1350348 from 9438149 the remainder is 8087801, and in adding 1350348 to it we get the sum with which we started, viz. 9438149, showing that an equal addition destroys an equal subtraction, and vice versa.

(2)	From	486478	(3)	5678473
	Take	197853		4789586

(4)	From	67890123	(5)	9734645	(6)	1494387
	Take	58917348		8887777		1285679

(7)	From	1946784	(8)	27684937	(9)	97648464
	Take	1553896		16795789		85639576

The scholar will here perceive that he has operated in addition and subtraction with mere numbers, without a name; but if the first question in addition meant dollars, the figures would be more expressive, and the result would be 25038 *dollars:* and if the second question expressed dollars and cents, the result would be \$1801 76: also, if the third question consisted of dollars, cents and mills, the result would be \$1672 03 9, which gives a practical meaning to the operation.

MULTIPLICATION AND DIVISION.

These rules have been called shorter methods of performing addition and subtraction.

All calculation consists in either increase or decrease; and the intention of all calculation is either to produce equality, or to determine the difference.

The distinction between the preceding two rules, and the present, may be thus stated:

Addition is / Multiplication is	increase	by *irregular* degrees. / by *regular* degrees.
Subtraction is / Division is	decrease	by *irregular* degrees. / by *regular* degrees.

In multiplication the number which is to be multiplied we call the *multiplicand*, that by which we multiply is known as the *multiplier*, and the result of the operation is the *product*.

RULE. Commence at the right hand and multiply each figure in the multiplicand by the multiplier, setting down and carrying as by the following example directed.

Ex. 1. Multiply 463 by 3 Operation 463
3
———
1389 Product.

Here we say 3 times 3 are 9, which being less than 10 set down 9; next 3 times 6 are 18, which is 8 units and 1 ten, set down the 8 units and carry the ten; then 3 times 4 are 12 and one from the preceding number makes 13; set down 13, being product of the last figure of the multiplicand, and the whole product is found to be 1389; which result may also be found by addition, thus 463
But we perceive it would be a tedious operation by addition if we should increase the same number thirty times, which could be done with ease by multiplication as above presented.

463
463
———
1389

Operate thus in all questions where the multiplier does not exceed 12,

2. Multiply 7432 by 5.

Operation.

```
  7432
     5
 37160 Product.
```

3. Multiply 14456 by 7.

```
 14456
     7
101192 Product.
```

4 Multiply 456789 by 12.

```
456789
    12
541468 Product.
```

5. Multiply 7893456 by 11.

```
 7893456
      11
85828016 Product.
```

If the multiplier exceed 12, multiply each figure in the multiplicand separately by each figure in the multiplier, commencing with the right hand figure of each and setting down the product directly under the multiplying figure; after each figure in the multiplier has been thus used add together the several products; the amount will be the required result.

1. Ex. Multiply 345678 by 254.

Operation.

```
  345678
     254
 1382712 product of 4 units.
1728390  product of 5 tens one place to the left hand.
691356   product of 2 hundreds one place farther to the left hand.
87802212 product of 345678 × 254.
```

2. Ex. Multiply 3456789012 by 432.

```
   3456789012
          432
   ----------
   6913578024
 10370367036
13827156048
-------------
1493332853184 Product.
```

The scholar is advised to recur to the operations and products of these several sums, which will be *proved* by a simpler process shown to him hereafter.

3	Ex.	Multiply	785678	by	246
4	"	"	"	"	7212
5	"	"	"	"	9027
6	"	"	"	"	2709
7	"	"	"	"	8811
8	"	"	"	"	99011
9	"	"	"	"	18009
10	"	"	"	"	80056
11	"	"	"	"	56008
12	"	"	"	"	70014.

Division is the reverse of multiplication; wherefore a question of multiplication may be proved by division, and and a question of division may be proved by multiplication. In division the sum to be divided is called the *dividend*, that by which we divide is the *divisor*, and the result of the division is called the *quotient*.

Example: How many 5's are contained in 15?

```
    from 15
subtract  5
    from 10
subtract  5
    from  5
subtract  5
          0
```

By three subtractions of 5 (the divisor) we find that 5 in 15 is contained three times, but this is a slow process of operation.

If the scholar is acquainted with his multiplication table and the analysis of it, he may easily divide any number when the divisor does not exceed 12, either by short or long division.

Ex. 1. Divide 465 by 3

Operation.

```
3)465(155 quotient.
  3
  --
  16
  15
  --
   15
   15
```

We say 3 in 4=1, set 1 in the quotient and $1\times3=3$ which subtracted from 4 leaves 1, bring down the next figure 6=16, then 3 in 16=5 and $3\times5=15$, which subtracted from16 leaves 1, take down 5=15 and 3 in 15=5, and 15 from 15=0.

By short division; 3 in 4=1 and 3 from 4 leaves 1, which is the tens to the next figure 6, wherefore now 3 in 16=5 and 1 remainder, which is the tens to the next 5=15 and 3 in 15=5. thus . . 3)465(155 quotient.

		Divide			Result
Ex.	2	345688	by	4	86422
"	3	123425	"	5	24685
"	4	326448	"	6	54408
"	5	78638	"	7	11234
"	6	356448	"	8	44556
"	7	726345	"	9	80705
"	8	853470	"	10	85347
"	9	671341	"	11	61031
"	10	726084	"	12	60507

Ex. 11 2624 by 64

Operation.

```
64)2624(41
   256
   ---
    64
    64
    --
     0
```

	Dividend	Divisor		Result.
Ex. 12 Divide	3456792 by	72	=	48011
" 13 "	7884 "	108	"	73
" 14 "	15359 "	29	"	$529\frac{18}{29}$

We see that here a remainder of 18 is left, which gives a result of $529\frac{18}{29}$, or, five hundred twenty-nine whole numbers and $\frac{18}{29}$ of a whole number, which is called a fraction; and as all fractions originate from the remainder of a division, this remainder placed above a horizontal line is called the numerator, because it denotes of how many parts the fraction consists; the number below the line is called the denominator, and shows into how many parts the whole number is divided; as $\frac{1}{2}$, (one half) $\frac{3}{4}$, (three fourths) $\frac{4}{5}$, (four fifths) $\frac{18}{29}$ (eighteen twenty-ninths); these fractions are called *simple* fractions; but such as $\frac{5}{2}$ (five halfs) $\frac{7}{4}$ (seven fourths), are called *improper* fractions, because the numerator is larger than the denominator, and dividing this numerator by the denominator we obtain what is called a *mixed* fraction, thus $5 \div 2 = 2\frac{1}{2}$, (two and a half) and $7 \div 4 = 1\frac{3}{4}$ (one and three fourths;) and finally $\frac{1}{2}$ of $\frac{3}{4}$, (one half of three fourths) $\frac{4\frac{1}{2}}{10}$ (four and one half tenths;) $\frac{7}{12\frac{1}{2}}$ (seven twelfths and a half,) these are called *compound* fractions; we have therefore simple fractions, improper fractions, mixed fractions, and compound fractions.

We have seen in the 14th. question of division, that we have obtained the result or quotient of $529\frac{18}{29}$, now by multiplying the whole number 529 by the denominator, and adding the numerator 18 thereto, we obtain $15359 \div 29$, being the whole numbers the same as before the divi-

sion was performed; we have therefore destroyed the effect of the division and proved at the same time that the division was correct.

GENERAL RULE OF MULTIPLICATION AND DIVISION OF WHOLE NUMBERS AND FRACTIONS.

Draw a perpendicular line and place multipliers and dividends on the right hand side of this line, and divisors on the left hand side; if fractions are found in the multipliers or divisors so placed, simplify them by multiplying the whole number by the denominator, and add thereto the numerator, and place the denominator on the opposite side of the perpendicular line and reduce them by *contraction* and *cancellation* to their lowest terms for the answer, as follows; cross out all equal numbers of ciphers found on opposite sides of the line, also all equal or similar numbers; next try if any of the opposing numbers have common factors, and if so, divide by these factors, and write the quotient in place of the original numbers; now see if any of your present numbers are themselves factors of opposing numbers, if so cross out entirely the smaller number, and substitute its quotient for the larger one; if we have now carried out our instructions faithfully our statement will be in its most condensed (and therefore simple) condition, and we proceed to *multiply* together the numbers on the right side of the line for a dividend, and those on the left for a divisor, and the result of this final division is the answer.

Statement.

320	400
11	5
5	33
126	189

Solution.

$4\ \cancel{3}\cancel{2}\cancel{0}$	$\cancel{4}\cancel{0}\cancel{0}\ 5$
$\cancel{1}\cancel{1}$	$\cancel{5}$
$\cancel{5}$	$\cancel{3}\cancel{3}\ \cancel{3}$
$2\ \cancel{6}\ \cancel{4}\cancel{2}\ \cancel{1}\cancel{2}\cancel{6}$	$\cancel{1}\cancel{8}\cancel{9}\ \cancel{2}\cancel{7}\ 9$
8	$45=5\frac{5}{8}$ Result.

Again, let us suppose it should be required, 1st. to multiply 91 by 72, the product to be divided by 43, and the quotient by 26.

And 2d. to multiply 67 by 59, the product to be divided by 15, and that quotient by 28, after which both quotients thus found to be multiplied together for the answer.

By way of exposition of the system now in use, we will perform the operation by it, as the question requires.

At first. 91
72
182
637
$6552 \div 43 = 152\frac{16}{43} \div 26 = \frac{5962}{1118} = 5\frac{37}{43}$ first quot.
43   130
255   22
215
102
86
16

And 2d. 67
59
603
355
$3953 \div 15 = 263\frac{8}{15} \div 28 = 9\frac{173}{420}$ 2d quotient.
30   252
95   11
90
53
45
8

And 3d. $5\frac{37}{43}$ first quotient.

$$\begin{array}{l} 5\frac{37}{43} \text{ first quotient.} \\ \underline{9\frac{173}{420}} \text{ second quotient.} \\ 45 \\ 2\frac{25}{420} \\ 7\frac{32}{43} \\ \underline{\frac{6401}{18060}} \\ 55\frac{34}{215} \text{ Answer.} \end{array}$$

Observe, that the above exhibition does not show all the figuring required by the usual manner of calculation; multiplying the two quotients, which are mixed fractions, will be found a most laborious operation, and will contrast strongly with our simple mode of solution as shown below.

No. 2 Statement. Solution.

$$\begin{array}{r|l} 43 & 91 \\ 26 & 72 \\ 15 & 67 \\ 28 & 59 \end{array} \qquad \begin{array}{r|l} 43 & \cancel{9}\cancel{1}\ \cancel{1}\cancel{3} \\ \cancel{2}\ \cancel{2}\cancel{6} & \cancel{7}\cancel{2}\ \cancel{1}\cancel{8}\ 3 \\ 5\ \cancel{1}\cancel{5} & 67 \\ \cancel{7}\ \cancel{2}\cancel{8} & 59 \\ \hline 215 & 11859 = 55\frac{34}{215} \end{array}$$

No. 3. It is required to multiply $4\frac{3}{4}$ by $\frac{2}{3}$ of $\frac{3}{4}$

Statement. Fractions resolved. Reduced to lowest term.

$$\begin{array}{|l} 4\frac{3}{4} \\ \frac{2}{3} \\ \frac{3}{4} \end{array} \qquad \begin{array}{r|r} 4 & 19 \\ 3 & 2 \\ 4 & 3 \end{array} \qquad \begin{array}{r|l} 4 & 19 \\ \cancel{3} & \cancel{2} \\ 2\ \cancel{4} & \cancel{3} \\ \hline 8 & 19 = 2\frac{3}{8} \end{array}$$

No. 4. Again multiply $2\frac{1}{3}$ by $1\frac{1}{7}$, and again by $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$.

Statement. Fractions resolved. Reduced to lowest term.

$$\begin{array}{|l} 2\frac{1}{3} \\ 1\frac{1}{7} \\ \frac{1}{2} \\ \frac{2}{3} \\ \frac{3}{4} \end{array} \qquad \begin{array}{r|l} 3 & 7 \\ 7 & 8 \\ 2 & 1 \\ 3 & 2 \\ 4 & 3 \end{array} \qquad \begin{array}{r|l} \cancel{3} & \cancel{7} \\ \cancel{7} & \cancel{8} \\ \cancel{2} & 1 \\ 3 & 2 \\ \cancel{4} & \cancel{3} \\ \hline 3 & 2 = \frac{2}{3} \text{ Ans.} \end{array}$$

No. 5 Suppose it is required to divide $\frac{1}{5}$ of 19 by $\frac{2}{3}$ of $\frac{3}{4}$.

Statement. Fractions resolved. Reduced to lowest term.

$$
\begin{array}{r|l}
\frac{2}{3} & \frac{1}{5} \\
\frac{3}{4} & 19
\end{array}
\qquad
\begin{array}{r|l}
5 & 1 \\
 & 19 \\
2 & 3 \\
3 & 4
\end{array}
\qquad
\begin{array}{r|l}
5 & 1 \\
 & 19 \\
\cancel{2} & \cancel{3} \\
\cancel{3} & \cancel{4}\ 2 \\
\hline
5 & 38 = 7\frac{3}{5} \text{ Ans.}
\end{array}
$$

No. 6. Again divide $3\frac{1}{6}$ by $9\frac{1}{2}$.

Statement. Fractions resolved. Reduced to lowest term.

$$
\begin{array}{r|l}
9\frac{1}{2} & 3\frac{1}{6}
\end{array}
\qquad
\begin{array}{r|l}
6 & 19 \\
19 & 2
\end{array}
\qquad
\begin{array}{r|l}
3\ \cancel{6} & \cancel{19}\ 1 \\
\cancel{19} & \cancel{2} \\
\hline
3 & 1 = \frac{1}{3} \text{ Ans.}
\end{array}
$$

No. 7. Divide $\frac{2}{3}$ of $\frac{1}{3}$ by $\frac{5}{7}$ of $7\frac{3}{5}$.

Statement. Fractions resolved. Reduced to lowest term.

$$
\begin{array}{r|l}
\frac{5}{7} & \frac{2}{3} \\
7\frac{3}{5} & \frac{1}{3}
\end{array}
\qquad
\begin{array}{r|l}
3 & 2 \\
3 & 1 \\
5 & 7 \\
38 & 5
\end{array}
\qquad
\begin{array}{r|l}
3 & \cancel{2} \\
3 & 1 \\
\cancel{5} & 7 \\
19\ \cancel{38} & \cancel{5} \\
\hline
171 & 7 = \frac{7}{171} \text{ Ans.}
\end{array}
$$

It is a fundamental principle that equal multiplication and division cannot effect any change in proportion; thus if we multiply 20 by 4 we obtain 80, and if we divide this product again by 4 we have 20 for the quotient, being the same as at first. Therefore, if in any calculation we find a multiplier and divisor having a common factor, we may *condense* these opposing terms by cancellation, and obtain the same result as if we had used the original numbers; and thereby save much labor and have additional assurance of correctness: this condensation is in fact exchanging a compound for an ingredient, and may be called *numerical analysis*.

Suppose it is required to analyze 126 to its original

*prime** factors: according to the *token* given by the properties of numbers, we see at a glance that it is an even number, and that 2 is factor, also the two left hand figures are the double of the right hand, (the token of 7,) also that the figures added horizontally give 9 (the token of 9;) to analyze 126 we would proceed thus:

```
 126
(2|63
  (7| 9
     3|3
```

therefore 2, 3, 3, 7 are the prime factors of the number 126, because it has originated from the multiplication of these 4 prime factors, as $2 \times 7 \times 3 \times 3 = 126$. If we now look at the next number to it, 127,† we will not find any token given by the properties, because it is a prime number, and cannot be analyzed or divided by any other number than 1.

It will be observed that no distinction is made between division of whole numbers and division of fractions, because our simple rule combines them.

REDUCTION

Teaches us to express identical quantities by different names: thus 1 dollar, 10 dimes, 100 cents, 1000 mills, are all distinct names for the same amount.

RULE.—Read the question carefully, and as soon as you discover the *name* of the *term required*, write down its sign (as an index of the answer) on the left side of the perpendicular line at the top, and the term or quantity for which an equality or new expression is sought on the right or opposite side of the line—this is our first step; the next is to write in the second place on the left, the term of the

*A *prime* factor is a number which can be divided perfectly only by the figure 1—a *compound* factor is a number divisible by larger factors than 1.

† See analytical table of Factors—Part II. of this volume.

same name with that of the first place on the right, and set opposite to it its equivalent in the next denomination, as shown in the tables of money, weights and measures, at the end of this volume; thus go on until you obtain a term on the right hand of the same *name* as that at the top of the left, our statement will then be complete—and we proceed forthwith to cancel opposing numbers, and then to multiply the remaining numbers on the left, to form a divisor, and those on the right for a dividend, the quotient of which will be the answer.

Example 1st. Reduce 20 tons to lbs.

Statement.

lbs.	20 tons.
ton 1	20 cwt.
cwt. 1	112 lbs.

Solution.

	20
	20
	112
	44800 Answer.

or 20 tons are equal to 44800 lbs.

Having no divison in this question, we multiply the factors of the right hand side for the answer, and to do this surely and easily we must be familiar with our "Easy Multipliers and Divisors." (Part II.)

Ex. 2. Reduce 44800 lbs. to tons.

Statement.

tons.	44800 lbs.
lbs. 112	1 cwt.
cwt. 20	1 ton.

Solution.

	$\cancel{448}\cancel{00}$ $\cancel{4}\cancel{0}=20$ Ans.
$\cancel{112}$	
$\cancel{20}$	

Ex 3. Reduce $\frac{5}{6}$ of a penny to the fraction of a £.

Statement.

£.	$\frac{5}{6}$ d.
d. 12	1 sh.
sh. 20	1 £.

Solution.

6	$\cancel{5}$
12	1
4 $\cancel{20}$	1
288	$1=\frac{1}{288}$ Answer.

Ex. 4th. Reduce $\frac{1}{288}$ of a £ to the fraction of a penny.

Statement.

$$\begin{array}{r|l} \text{d.} & \frac{1}{288}\ \text{£.} \\ \text{£. } 1 & 20\ \text{sh.} \\ \text{sh. } 1 & 12\ \text{d.} \end{array}$$

Solution.

$$\begin{array}{r|l} 6\ \cancel{24}\ \cancel{288} & 1 \\ & \cancel{20}\ 5 \\ & \cancel{12} \\ \hline 6 & 5 = \frac{5}{6}\ \text{Answer.} \end{array}$$

Although the solution is performed with abstract or nameless numbers, the result of which is the equality required by the question, we can always find the name of the answer by the index at the top of the left hand side of the statement. If fractions are found in your statement, reduce them to whole numbers as already taught.

Ex. 5th. Reduce $\frac{5}{9}$ of a cent to the fraction of a dollar.

Statement.

$$\begin{array}{r|l} \$ & \frac{5}{9}\text{cent} \\ 100 & 1\$ \end{array}$$

Solution.

$$\begin{array}{r|l} 9 & \cancel{5} \\ 20\ \cancel{100} & 1 \\ \hline 180 & 1 = \frac{1}{180}\ \text{Answer.} \end{array}$$

Ex. 6th. Reduce $\frac{1}{180}$ of a dollar to the fraction of a cent.

Statement.

$$\begin{array}{r|l} \text{cents} & \frac{1}{180} \\ \$1 & 100\ \text{cts.} \end{array}$$

Solution.

$$\begin{array}{r|l} 9\ \cancel{180} & 1 \\ 1 & \cancel{100}\ 5 \\ \hline 9 & 5 = \frac{5}{9}\ \text{Answer.} \end{array}$$

Ex. 7th. Reduce $\frac{2880}{7}$ of a farthing to the fraction of a £.

Statement.

$$\begin{array}{r|l} \text{£} & \frac{2880}{7}\ \text{farth.} \\ \text{farth. } 4 & 1\text{d.} \\ \text{d. } 12 & 1\text{sh.} \\ \text{sh. } 20 & 1\text{£.} \end{array}$$

Solution.

$$\begin{array}{r|l} 7 & \cancel{2880}\ \cancel{24}\ \cancel{6}\ 3 \\ \cancel{4} & \\ \cancel{12} & \\ \cancel{20} & \\ \hline 7 & 3 = \frac{3}{7}\ \text{£ Answer.} \end{array}$$

Ex. 8th. Reduce $\frac{3}{7}$ of a £ to the fraction of a farthing.

Statement.		Solution.	
farth.	$\frac{3}{7}$£.	7	3
£ 1	20sh.		20
sh. 1	12d.		12
d. 1	4f.		4
		7	2880 $=\frac{2880}{7}$ Answer.

Ex. 9th. Reduce $\frac{10}{11}$ of a minute to the fraction of a day.

Statement.		Solution.	
day	$\frac{10}{11}$ minute	11	$1\cancel{0}$
minutes 60	1 hour	$6\cancel{0}$	1
hour 24	1 day	24	1
		1584	$1=\frac{1}{1584}$ Answer.

Ex. 10th. Reduce $\frac{1}{1584}$ of a day to the fraction of a minute.

Statement.		Solution.	
minute	$\frac{1}{1584}$ day	11 $\cancel{264}$ $\cancel{1584}$	1
day 1	24 hours	1	$\cancel{24}$
hour 1	60 minutes	1	$\cancel{6}0$
		11	$10=\frac{10}{11}$ Ans.

To reduce money, weight or measure to a higher or lower denomination, we refer to their respective tables.

Ex. 1st. Reduce 375 £ to its lowest denomination of shillings.

Statement.		Solution.
sh.	375	375
1	20sh.	20
		7500 Answer.

Ex. 2d. If the shillings must be reduced to the denomination of pence, we have

Statement.		Solution.
d.	7500sh.	7500
sh. 1	12d.	12
		90000 Answer, or

the proportion 375 £=90000 d. also,
the proportion 375 £=7500 shillings.

Ex. 3d. Reduce 9460 ounces to lbs.

Statement.		Solution.
lb.	9460oz.	4 $\cancel{16}$ | $\cancel{9460}$ 2365
oz. 16	1lb.	lb. oz.
		4 | 2365(591 4 Ans.

Ex. 4th. Reduce 591 lbs. 4 ounces to quarters.

Statement.		Solution.
quarter	591lbs.	| 591
28 lbs.	1quarter	28 |
		28 | 591(21qr. 3lb. 4oz. [Ans.
		56
		31
		28
		3

Ex. 5th. Reduce 21 quarters 3 lb. 4 oz. to cwt.

Statement.		Solution.
cwt.	21quarters	4 | 21 quarters
4	1cwt.	cwt. qr. lb. oz.
		4 | 21=5 1 3 4
		20
		1

To reduce fractions to compound numbers or to their proper value, and the proper value to fractions, we also refer to the tables, thus:

Ex. 1st. Reduce $\frac{2}{3}$ of a £ to shillings.

Statement.

$$\begin{array}{r|l} \text{sh.} & \frac{2}{3}\text{£} \\ 1 & 20\text{sh.} \end{array}$$

Solution.

$$\begin{array}{r|l} 3 & 2 \\ & 20 \\ \hline 3 & 40 = 13\frac{1}{3} \text{ Answer.} \end{array}$$

Ex. 2d. Now reduce $\frac{1}{3}$ of a shilling to pence.

Statement.

$$\begin{array}{r|l} \text{d.} & \frac{1}{3}\text{sh.} \\ 1 & 12\text{d.} \end{array}$$

Solution.

$$\begin{array}{r|l} \cancel{3} & 1 \\ & \cancel{12}\ 4 \text{ Ans.} \end{array}$$

therefore $\frac{2}{3}$ £ = 13 sh. 4 d.

Compound numbers are reduced to fractions by operating in the contrary way.

Ex. 1st. Reduce 13 sh. 4 d. to the fraction of a £.

Statement.

$$\begin{array}{r|l} \text{d. } 12 & 4\text{d.} \\ \text{sh. } 20 & 13\text{sh.} \end{array}$$

Solution.

$$\begin{array}{r|l} 3\ \cancel{12} & \cancel{4}\ 1 = \frac{1}{3} \\ \hline \cancel{20} & \cancel{13\frac{1}{3}} \\ 3 & \cancel{40}\ 2 = \frac{2}{3}\text{£} \end{array}$$

Solution horizontally.

$$\frac{\cancel{4}\ 1}{\cancel{12}\ 3} \Big| \frac{13\frac{1}{3}}{20} \Big| \frac{40}{60} = \frac{2}{3}.$$

Placing against 4d. its denominator 12, and against 13sh. its denominator 20, we reduce $\frac{4}{12}$, to its lowest term $\frac{1}{3}$, bring it to 13 and reduce now $\frac{13\frac{1}{3}}{20}$ = to $\frac{40}{60} = \frac{2}{3}$£.

Ex. 2d. Reduce $\frac{13}{16}$ of a day to its proper value.

Statement.

$$\begin{array}{r|l} \text{hours} & \frac{13}{16} \text{ day} \\ 1 & 24 \text{ hours} \end{array}$$

Solution.

$$\begin{array}{r|l} 2\ \cancel{16} & 13 \\ & \cancel{24}\ 3 \\ \hline 2 & 39 = 19\frac{1}{2} \text{ or 19 hours 30 min.} \end{array}$$

Ex. 3d. Reduce 19 hours 30 min. to the fraction of a day.

Statement.

60	30 min.
24	19

Solution.

2 $\cancel{60}$	$\cancel{30}$ 1 $=\frac{1}{2}$
8 $\cancel{24}$	$\cancel{19}\frac{1}{2}$
2	$\cancel{39}$ 13
16	13 $=\frac{13}{16}$ Answer.

Ex. 4th. Reduce $\frac{2}{13}$ of a ton to its lowest terms of cwt. lb. oz. and drams.

Statement.

cwt.	$\frac{2}{13}$
1	20 tons
lb.	$\frac{1}{13}$cwt.
1	112lb.
oz.	$\frac{8}{13}$lb.
1	16oz.
drams	$\frac{11}{13}$oz.
1	16
1	

Solution.

13	2
	20
13	$40=3\frac{1}{13}$
13	1
1	112
13	$112=8\frac{8}{13}$
13	8
1	16
13	$128=9\frac{11}{13}$
13	11
1	16
13	$176=13\frac{7}{13}$

Result, 3cwt. 8lb. 9oz. $13\frac{7}{13}$dr.

Ex. 5th. Reduce 3cwt. 8lb. 9oz. $13\frac{7}{13}$drs. to the fraction of a ton.

Statement.

16	$13\frac{7}{13}$
16	9
112	8
20	3

Solution.

$\cancel{16}$	$13\frac{7}{13}=\frac{176}{13}$
13	$\cancel{176}$ 11 or $\frac{11}{13}$
$\cancel{16}$	$9+\frac{11}{13}=\frac{128}{13}$
13	$\cancel{128}$ 8 or $\frac{8}{13}$
$\cancel{112}$	$8+\frac{8}{13}=\frac{112}{13}$
13	$\cancel{112}$ 1 or $\frac{1}{13}$
$\cancel{20}$	$3+\frac{1}{13}=\frac{40}{13}$
13	$\cancel{40}$ 2
13	$2=\frac{2}{13}$ Answer.

See next page.

Illustration—$13\frac{7}{13}=\frac{176}{13}$ we cancel 16 into 176 and get 11 which$=\frac{11}{13}$ this fraction we carry to the next denomination, viz. 9oz. and say $\frac{11}{13}$ and $9=\frac{128}{13}$ and 16 against 128 gives 8 making $\frac{8}{13}$ which we carry to the lbs. and say $8+\frac{8}{13}=\frac{112}{13}$ and get $\frac{1}{13}$ which carried to the cwt. $3+\frac{1}{13}=\frac{40}{13}$ which gives us $\frac{2}{13}$ of a ton.

Solution horizontally.

$$\frac{13\frac{7}{13}=\overset{11}{\cancel{176}}}{\cancel{16}\qquad 13}\Bigg|\frac{9\frac{11}{13}=\overset{8}{\cancel{128}}}{\cancel{16}\qquad 13}\Bigg|\frac{8\frac{8}{13}=\overset{1}{\cancel{112}}}{\cancel{112}\qquad 13}\Bigg|\frac{3\frac{1}{13}=\overset{2}{\cancel{40}}}{\cancel{20}\qquad 13}=\frac{2}{13}\text{ ton. Ans.}$$

How to reduce compound fractions to single ones, has been taught by multiplication and division.

Ex. 1st. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ to its simplest expression or lowest term.

Statement.

$$\left|\begin{array}{l}\frac{2}{3}\\ \frac{3}{4}\\ \frac{5}{6}\end{array}\right.$$

Solution.

$$\begin{array}{r|l}\cancel{3} & \cancel{2}\\ 2\ \cancel{4} & \cancel{3}\\ 6 & 5\\ \hline 12 & 5=\frac{5}{12}\text{ Answer.}\end{array}$$

Ex. 2d. Or if we have to divide $34\frac{1}{2}$ by 46, its lowest expression will be

Statement.

$$46|34\tfrac{1}{2}$$

Solution.

$$\begin{array}{r|l}2\ \cancel{46} & \cancel{34\frac{1}{2}}\\ 2 & \cancel{69}\ 3\\ \hline 4 & 3=\frac{3}{4}\text{ Ans. single frac.}\end{array}$$

Reduce $\dfrac{2\frac{1}{2}}{3}$ of $\dfrac{6\frac{3}{4}}{1\frac{1}{2}}$ to a single fraction. Ans. $3\frac{3}{4}$.

" 3cwt. 2qrs. 14lbs. to the fraction of a ton. Ans. $\frac{29}{160}$.

" $12\frac{1}{2}$ cents to the fraction of a dollar. Ans. $\frac{1}{8}$.

" $\frac{1}{2}$ of $\frac{2}{3}$ of $5\frac{3}{8}$ of $\dfrac{7\frac{5}{9}}{54}$ to a single fraction. Ans. $\frac{731}{972}$.

ADDITION AND SUBTRACTION OF FRACTIONS.

☞ In *fractions*, the *order* of the fundamental rules of increase and decrease is reversed; because, except in very simple cases, we must *multiply* imperfect quantities before we can *add* them.

These operations are easily performed, if the fractions have common denominators. Examples, $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$ added together equal $\frac{6}{7}$: here we merely add the numerators, and place their common denominator under the sum of the numerators for the answer.

Again, add $\frac{1}{9}$, $\frac{4}{9}$, $\frac{7}{9}$, $\frac{5}{9}$, $\frac{8}{9}$ together$=\frac{25}{9}=2\frac{7}{9}$; adding the numerators we have 25, placing the common denominator 9 under 25, we have the improper fraction $\frac{25}{9}$, or 2 whole numbers or integers and $\frac{7}{9}$, making the mixed fraction $2\frac{7}{9}$.

To add or subtract fractions not having a common denominator, bring the different denominators to a common "multiple," or the least common denominator, and raise the different numerators by the common denominator in the same proportion, (as is shown on the next page) after which add as above.

To bring different fractions to a common denominator, the largest denominator is retained: but all other denominators which, according to the Properties of figures and numbers, can be resolved into any common factor contained in the largest denominator or any other denominator which is retained, are cancelled or thrown out of the question; after which multiply the remaining denominators or figures or numbers to a continued product, for the least common denominator.

Example. Find the least common denominator or multiple in the following series of denominators, from 2 to 10 inclusive, to wit: 2, 3, 4, 5, 6, 7, 8, 9, 10. 10 being the largest denominator is retained; 9 has no factor in common with 10, therefore 9 is retained; 8 has a relation or factor in common with 10, to wit, 2: which cancelled into 8 leaves

4, therefore 4 is set down; 7 has no relation or factor in common with 10, 9 or 4, therefore 7 is retained; 6 has two factors, 2 and 3—the two is contained or cancelled in 10, and the three in 9, therefore 6 is left out; 5 is contained or cancelled in 10, therefore 5 is left out; 4 is contained or cancelled in 4 therefore 4 is left out; 3 is contained or cancelled in 9, therefore 3 is left out; 2 is contained or cancelled in 4 or 10, therefore 2 is left out.

Now the numbers retained or remaining, to wit, 10. 9. 4. 7. multiplied to a continued product, form the least common denominator of the above numbers, namely, 2520, by which raise the numerators in the same proportion, that is, by dividing the common multiple by each denominator, and multiplying the quotient by the numerator of the respective denominators; and this done, then add as previously.

We have already said that in proceeding to add fractions having different denominators, we must first secure their *least* common *multiple;* to accomplish this important preliminary in the most perfect and safe manner is certainly desirable: the following directions for this purpose are given by one of our ablest American Arithmeticians.

"Required the least common multiple of 12, 25, 30 and 45? (Lewis's Arithmetical Expositor, p. 91.)

3	12	25	30	45
5	4	25	10	15
2	4	5	2	3
	2	5	1	3

and $3\times 5\times 2\times 2\times 5\times 3=900$, is the common multiple required."

Let us contrast the above operation with our system: we wish to find the least common multiple of 12, 25, 30, 45—according to the general directions previously given, we say 45 is the largest number; 30 has a common factor with 45, viz. 15, therefore we divide 30 by this factor and say $30\div 15=2$ and place this quotient instead of 30; 25 has also a factor in common with 45, viz. 5—we cancel 25 by 5; we

then find that 12 and 45 have a common factor in 3, so 12 $\div 3=4$; we now see that the substitute for 30, viz. 2, is contained in this last factor, we therefore cancel *it* and our statement will stand thus:

$$\cancel{12},\quad \cancel{25},\quad \cancel{30},\quad 45\times 4\times 5=900 \text{ as above.}$$
$$4 \quad\ 5 \quad\ \cancel{2} \quad \text{new factors.}$$

It is frequently desirable to find a common factor or measure for several different numbers—the following is the rule laid down for that purpose by the celebrated *Emerson*, See N. Am. Arithmetic, p. 22 and 23.

"Divide the greater number by the smaller and this divisor by the remainder, and thus continue dividing the last divisor by the last remainder till nothing remains; the divisor last used will be the number required. Example.—What is the greatest common measure of 918, 1998, and 522?

```
918)1998(2                    54)522(9
    1836                         486
    ----                         ---
     162)918(5                    36)54(1
         810                         36
         ---                         --
         108)162(1                   18)36(2
             108                        36
             ---
              54)108(2
                 108                 Ans. 18."
```

We will now apply our system to this question: 918, 1998, 522 are each numbers the sum of whose figures added horizontally can be measured by 9, *therefore* 9 is a factor or measure in these numbers themselves, so we see also that 2 is a factor in these numbers, (see List of Factors Part II.) and $9\times 2=18$ as above.

Ex. 1st. Add $\frac{1}{8}$, $\frac{3}{10}$, $\frac{4}{12}$, together.

2 $\quad \frac{1}{8}\times 15 = 15 \qquad$ (120 common multiple.

5 $\quad \frac{3}{10}\times 12=36$

$\quad \frac{4}{12}\times 10=40$

$\qquad\qquad \overline{91}$

$\qquad\qquad \overline{120}$ Answer.

To find the common multiple or the smallest common denominator, we say 12 is the largest denominator, therefore it is retained; making a comparison with 10, we find the common factor of 10 and 12 to be 2, we divide this 2 in 10=5 which we mark by small figures against 10. Comparing 8 and 12 the common factor is 4, dividing 4 in 8=2 marked by a small figure; now 2, 5, 12 multiplied together =120 the least multiple. The numerator now raised in the same proportion in dividing the denominator, 8 in 120=15 placing that quotient against the numerator and multiplied by 1=15, dividing now by the denominator, 10 in 120=12 × 3=36, then by the denominator, 12 in 120=10 × 4=40, adding these new numerators 15+36+40=91 and placing the common denominator 120 under it, $\frac{91}{120}$ is the answer.

Ex. 2d. Add $\frac{9}{10}$, $\frac{3}{8}$, $\frac{6}{7}$, $\frac{2}{7}$, $\frac{8}{21}$ together.

$$
\begin{array}{rl}
{}^{5}\ \frac{9}{10} \times 84 = 756 & \text{(840 Multiple.} \\
{}^{8}\ \frac{3}{8} \times 105 = 315 & \\
\frac{6}{7} \times 120 = 720 & \\
\frac{2}{7} \times 120 = 240 & \\
\frac{8}{21} \times 40 = 320 & \\
\overline{2351} & \\
\overline{840} = 2\frac{671}{840} & \text{Answer.}
\end{array}
$$

Ex. 3d. Add $9\frac{2}{7}$, $12\frac{5}{14}$, $\frac{4}{10}$, $\frac{3}{8}$, $21\frac{3}{4}$

$$
\begin{array}{rrl}
 & 9\frac{2}{7} \times 40 = 80 & \text{(280 multiple.} \\
{}^{14} & 12\frac{5}{14} \times 20 = 100 & \\
{}^{5} & \frac{4}{10} \times 28 = 112 & \\
{}^{4} & \frac{3}{8} \times 35 = 105 & \\
 & 21\ \frac{3}{4} \times 70 = 210 & \\
 & \overline{42} \qquad \overline{607} & \\
 & 2\frac{47}{280} \qquad \overline{280} = 2\frac{47}{280} & \text{carried to the whole} \\
 & \overline{44\frac{47}{280}} & \text{[numbers.}
\end{array}
$$

Ex. 4th. Add $19\frac{5}{12}$, $\frac{7}{8}$, $7\frac{1}{2}$, $3\frac{2}{3}$ together.

$$
\begin{array}{llll}
12 & 19\frac{5}{12} \times 2 = & 10 & \text{(24 multiple.} \\
2 & \frac{7}{8} \times 3 = & 21 & \\
 & 7\frac{1}{2} \times 12 = & 12 & \\
 & 3\frac{2}{3} \times 8 = & 16 & \\
 & \overline{29} & \overline{59} & \\
\text{Ans.} & \overline{31\frac{11}{24}} & \overline{24} = 2\frac{11}{24} & \text{carried to the whole numbers.}
\end{array}
$$

NOTE—If there are compound fractions given in a question they must first be reduced by multiplication or division, to a single fraction.

Ex. 5th. Add $\frac{5}{6}$ of $\frac{9}{10}$ and $\frac{7}{12}$ of $\frac{4}{5}$. In multiplying these compound expressions we obtain the single fraction, as

$$
\begin{array}{r|l}
2\ \cancel{6} & \cancel{5} \\
2\ \cancel{10} & \cancel{9}\ 3 \\
\hline
4 & 3 = \frac{3}{4}
\end{array}
\text{ single fraction.} \qquad
\begin{array}{r|l}
3\ \cancel{12} & 7 \\
5 & \cancel{4} \\
\hline
15 & 7 = \frac{7}{15}
\end{array}
\text{ single fraction}
$$

now these single fractions are to be added together, therefore

$$
\begin{array}{ll}
\text{add } \frac{3}{4} \times 15 = 45 & \text{(60 multiple.} \\
\text{and } \frac{7}{15} \times 4 = 28 & \\
\qquad \overline{73} & \\
\qquad \overline{60} = 1\frac{13}{60} & \text{Answer.}
\end{array}
$$

If fractions of a £. sh. d. or cwt. qr. and lbs. are given to be added together, they must first be reduced to their proper value.

Ex. 6th. Add $\frac{7}{8}$ of a £ to $\frac{3}{4}$ of a shilling.

$$
\begin{array}{r|l}
\text{First. sh.} & \frac{7}{8}\text{£.} \\
1 & 20\text{sh.}
\end{array}
\qquad
\begin{array}{r|l}
2\ \cancel{8} & 7 \\
 & \cancel{20}\ 5 \\
\hline
2 & 35 = 17\frac{1}{2}
\end{array}
\qquad
\begin{array}{rr}
\text{sh.} & \text{d.} \\
17 & 6 \\
 & 9 \\
\hline
18 & 3 \text{ Answer.}
\end{array}
$$

$$
\begin{array}{r|l}
\text{Second d.} & \frac{3}{4}\text{sh.} \\
1 & 12\text{d.}
\end{array}
\qquad
\begin{array}{r|l}
\cancel{4} & 3 \\
 & \cancel{12}\ 3 \\
\hline
 & 9\text{d.}
\end{array}
$$

Ex. 7th. From $\frac{7}{12}$ take $\frac{1}{4}$.

$$\begin{array}{l} \frac{7}{12}\times 1=7 \quad (12 \text{ multiple.} \\ \frac{1}{4}\times 3=\underline{3} \\ \qquad\quad 4 \\ \qquad\quad \overline{12}=\frac{1}{3} \text{ Answer.} \end{array}$$

Ex. 8th. From $9\frac{1}{5}$ take $4\frac{3}{4}$.

$$\begin{array}{l} 9\frac{1}{5}\times 4=\ \ 4 \quad (20 \text{ multiple.} \\ \underline{4\frac{3}{4}\times 5=15} \\ 4\ \frac{9}{20} \text{ Answer.} \end{array}$$

As we cannot subtract 15 from 4, we must borrow one which is equal to the multiple $20+4=24$ from which we subtract $15=\frac{9}{20}$, having borrowed 1 for the fraction we must pay it back to the whole number, and the result is $4\frac{9}{20}$.

Ex. 9th. From $7\frac{7}{8}$ take $3\frac{2}{3}$.

$$\begin{array}{l} 7\frac{7}{8}\times 3=21 \quad (24 \text{ multiple.} \\ \underline{3\frac{2}{3}\times 8=16} \\ 4 \qquad\quad \frac{5}{24} \text{ Answer. } 4\frac{5}{24}. \end{array}$$

Ex. 10th. From $\frac{5}{6}$ take $\frac{3}{5}$ of $\frac{5}{8}$. (See *note*, Ex. 4th.)

$$\begin{array}{l} \frac{5}{6}\times 4=20 \quad (24 \text{ multiple.} \\ \frac{3}{8}\times 3=\ \ \underline{9} \\ \qquad\quad \frac{11}{24} \text{ Answer.} \end{array} \qquad \begin{array}{r|l} \cancel{5} & 3 \\ 8 & \cancel{5} \\ \hline 8 & 3 \end{array}$$

Ex. 11th. From $\frac{3}{4}$ of a £. take $\frac{7}{24}$ of a shilling.

$$\begin{array}{r|l} £\ \cancel{4} & 3 \\ 1 & \cancel{20}\ 5 \\ & \overline{15} \text{ shillings.} \\ & \text{take } 3\frac{1}{2}\text{d.} \\ & \overline{14\ \ 8\frac{1}{2}\text{d. Answer.}} \end{array} \qquad \begin{array}{r|l} \cancel{s}\ \cancel{24} & \cancel{7}\ \ 3\frac{1}{2}\text{ d.} \\ 1 & \cancel{12} \end{array}$$

QUESTIONS FOR EXERCISE.

Ex. 12.	Add $5\frac{2}{3}$, $6\frac{7}{8}$ and $4\frac{1}{2}$ together.	Ans.	$17\frac{1}{24}$
" 13.	" $\frac{7}{10}$, $\frac{11}{12}$ and $\frac{4}{9}$ "		$2\frac{11}{180}$
" 14.	" $\frac{3}{4}$ of $\frac{7}{9}$, and $\frac{4}{5}$ of $\frac{5}{9}$ "		$1\frac{1}{36}$
" 15.	" $\frac{2}{5}$, $\frac{5}{8}$ and $\frac{3}{7}$ "		$1\frac{101}{280}$
" 16.	" $\frac{1}{3}$ of 95, and $\frac{7}{8}$ of 14 together.		$43\frac{11}{12}$
" 17.	" $\frac{3}{4}$ of a mile to $\frac{7}{10}$ of a furlong		6 fur. 28 p.
" 18.	" $\frac{7}{8}$ of a mile, $\frac{2}{3}$ of a yard, and $\frac{3}{4}$ of a foot		7 furl. 2 ft. 9 in.
" 19.	From $\frac{1}{3}$ of 76 take $\frac{3}{4}$ of 21		$9\frac{7}{12}$
" 20.	" $96\frac{1}{3}$ take $14\frac{3}{7}$		$81\frac{19}{21}$
" 21.	" $\frac{109}{110}$ take $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$		$\frac{163}{220}$
" 22.	" $14\frac{1}{4}$ take $\frac{2}{3}$ of 19		$1\frac{7}{12}$

GENERAL RULE OF SIMPLE PROPORTION.

The general rule of simple proportion differs very little from that of *reduction;* it teaches to find from *three* given terms or numbers, a proportional *fourth* term or number; whereas reduction from *one* term given, requires a *second* term for a proportion or equality.

Simple proportion is one of the most important agents of the science of calculation.

A question in simple proportion gives 3 terms, by which a 4th term is found by calculation for the answer; these four terms are called a *Proportional,* of which the 1st and 4th are called the extremes, and the 2d and 3d terms the means, which must have the following *properties, viz. The product of the extremes must be equal to the product of the means.*

Example. If two apples cost 4 cents, how many cents will 40 apples cost?

Stated thus in the usual single rule of three.

apples.		cents.		apples.
2	:	4	::	40
				4

2)160=80 Ans. or 4th. term.

multiplying the 2d and 3d terms together, and dividing that product by the 1st term, we have the answer or the fourth term, thus: 2 : 4 :: 40 : 80 which have the properties required, thus:

Extremes $2\times 80=160$
Means $4\times 40=160$

Ex. 2d. If 3 yards cost 7 dollars, how many dollars must be paid for 21 yards?

Statement in the usual rule.

yds.		$		yds.
3	:	7	::	21
				7

3)147= $49 Answer.

Which gives the proportional 3 : 7 :: 21 : 49, and the properties required.

On those fundamental principles and properties, the statement by the *Chain Rule** would stand thus:

$	21 yds.
3	$7

Here we may see that those numbers which must be multiplied together, stand on the right hand side of the line for multipliers, which product must be divided by the 1st. term, and will give the same result; but by this general form of statement, we link the proportions or equalities together for easy and correct calculation, by contraction and

* This manner of statement has been long in use on the continent of Europe, especially in Germany, and the Netherlands—it is called the *chain rule* because its terms are written in *links*.

cancellation, even in questions involving different conditions.

Ex. 3d. If 1 pint cost 10d. how many £. will 3hhds. cost.

Statement.		Solution.	
£.	3hhds.		$\cancel{3}$
hhd. 1	63galls.		63 Answer.
gall. 1	4quarts.	$\cancel{4}$ $\cancel{12}$	$\cancel{4}$
quart 1	2pts.	$\cancel{20}$	$\cancel{2}$
pint 1	10d.		1$\cancel{0}$
d. 12	1sh.		
sh. 20	1£.		

Here the conditions of the question are, if 1 pint cost 10 *pence*, how many £. will three hhds. cost?

Our first term is 3 hhds. and as *hhd.* is on the right hand side of the line, we must begin on the left hand with hhds. and set down on the right hand its equivalent, and so on until we come to the antecedent of the condition, which is here 1 pint, set it down on the left hand, and 10d. the consequent, on the right hand side of the line, but as pence is not the same name as £, (the answer,) we go on by its table as before, until the *name* of the answer is found, and our statement is performed and ready to be contracted and cancelled to its lowest term.

The statement by the usual rule stands thus:

1pt. : 10d. :: 3hhds.

```
          63
         ---
         189
           4
         ---
         756
           2
        ----
        1512
          10
    12)15120
    20)1260
       ----
          63 Ans. in £.
```

We see here that we have to reduce the question to its lowest term or answer, by multiplication and division; whereas our plan of proceeding naturally brings the original numbers together ready for cancellation.

Let us suppose the same question, with additional conditions.

Ex. 4th. Received from England, 3 hhds. of liquor, at 10d. sterling per pint, according to invoice, for which we had to pay for freight and duty 25 per cent., exchange being at 10 per cent. premium; how many dollars will the 3 hhds. cost us here in Philadelphia.

Statement.			Solution.	
$	3hhds.			$\cancel{3}$
1	63 gals.		$\cancel{4}$ $\cancel{12}$	$\cancel{63}$ 7
1	4 quarts.		$\cancel{20}$	$\cancel{4}$
1	2 pints.		$\cancel{9}$	$\cancel{2}$
1	10d.			1$\cancel{0}$
12	1sh.		$\cancel{100}$	$\cancel{40}$
20	1£.		$\cancel{25}$ $\cancel{100}$	$\cancel{125}$ 5
9	40$			11$\cancel{0}$
100	125 duty	additional		$385 Ans.
100	110 premium	conditions.		

By the old system we should require four distinct calculations, thus stated:

As	1pt.	:	10d.	::	3hhd.	:	63£.	
"	9£.	:	$40	::	63£.	:	$280	
"	$100	:	$125	::	$280	:	$350	
"	$100	:	$110	::	$350	:	$385	Answer.

It is plain that after our statement of the rule of *Reduction* and *Simple Proportion*, it would be superfluous to have particular rules for *Barter*, *Practice*, *Loss and Gain*, and *Exchange*; since our general rules contain directions for solving all such questions.

Ex. 5th. A has 90 yards of linen worth 4sh. 6d. per yard, which he will exchange with B for muslin at 2sh. per yard, how many yards of muslin must A receive.

Statement.

yard muslin	90 yards linen
yard linen 1	$4\frac{1}{2}$ sh.
sh. 2	1 yard muslin

Solution.

	$\cancel{90}$ 45
$\cancel{2}$	9
2	
2	$405 = 202\frac{1}{2}$ Ans.

Ex. 6th. How many hogsheads of molasses at 3sh. per gallon, must I give for 7 tuns of wine at 10d. per pint.

Statement.

hhd.	7 tuns of wine.
1	2 pipes.
1	2 hhds.
1	63 gallons.
1	4 quarts.
1	2 pints.
1	10d.
12	1sh.
3	1 gallon.
63	1 hhd.

Solution.

3 $\cancel{12}$	7
	2
3	2
	$\cancel{63}$
$\cancel{63}$	
	$\cancel{4}$
	2
	10
9	$560 = 62\frac{2}{9}$ Ans.

Ex. 7th. Sold 5 cwt. 1 qr. of sugar, at 850 cts. per cwt. for which I received in pay 24 yards of cloth; how many cents did it cost per yard?

Statement.

cts.	1 yard cloth.
yd. cloth 24	$5\frac{1}{4}$ cwt. sugar.
cwt. 1	850 cts.

Solution.

2 $\cancel{4}$	$\cancel{21}$ 7
8 $\cancel{24}$	$\cancel{850}$ 425
16	$2975(185\frac{15}{16}$ cts.

Ex. 8th. A received of B, a quantity of broad cloth which he had sold for 450 cts. but charged to A at 500 cts. In payment, A let B have a quantity of wheat which he had sold to others at 50 cts. per bushel, what must A charge for his wheat, to meet the advance B made on his broad cloth?

Statement.

exchange price cts.	150 cts. money price.
money price 450 cts.	500 exchange price.

Solution.

$\cancel{450}$	$\cancel{150}$
3	500
3	500(166⅔

Ex. 9th. Bought 12 gross of penknives at $9 per gross, and paid for them in cloth at 3sh. 6d. per yard, New York currency, how many yards will be required to pay for the knives?

Statement.

yard	12 gross penknives.
1	$9
1	8sh. N. Y. currency.
3½	1 yard cloth.

Solution.

	12
	9
7	8
	2
7	1728(246 6/7 Ans.

Ex. 10th. Reduce £783 English money to Federal money at par.

Statement.

$	783£.
£ 9	$40

Solution.

	$\cancel{783}$ 87
$\cancel{9}$	40
	$3480 Ans.

Ex. 10th. Reduce the same at 10 per cent. premium.

Statement.

$	£783
£. 9	$40
$100	110 prem.

Solution.

	$\cancel{783}$ 87
$\cancel{9}$	$\cancel{40}$ 2
$\cancel{5}$ $\cancel{100}$	$\cancel{110}$ 22
	3828 Ans.

Ex. 12th. If a merchant of New York has to pay in England the sum of £783 at 10 per cent. premium, he must send $3828 or pay the same to a broker to obtain a draft for it; if we now suppose (as was the case lately) that New York funds are 6 per cent. better than those of Philadelphia, and that a Philadelphia merchant had received the same merchandise from England, how much must he pay the Broker?

Statement.

$	£783
£ 9	$40
$100	$110 premium
$100	106 of Phila.

Solution.

	~~783~~ 87
~~9~~	~~40~~ 2
~~$~~ ~~100~~	~~110~~ 22
100	106
100	4057 68 Ans.

the merchant of New York pays 3828

229 68 diff.

From this we have the conclusion that the merchant of New York can sell the same article for the same price and gain on the whole transaction $229 68 more than the merchant of Philadelphia, which illustrates plainly the influence of exchange on commerce of different states in the union, as also of the nations of the world in general.

It is evident by this method of statement, if a question gives particular and additional conditions, that we are not obliged as in usual systems, to make separate statements and operations; we set all the conditions as fixed, underneath, and cancel as usual.

Suppose A buys 742lbs. of wool on the following condition, to reduce 5 per cent. tret (*tret* means an allowance for waste in commodities) and to pay for 1lb. net weight 9 shillings Pennsylvania currency, at 6 per cent. discount; he sells this wool again to B on the same condition, with a profit of 20 per cent. how much must B pay to A in federal money.*

*This same question, in *New York Currency*, is taken out of my

$	742 lb. gross weight.	$\cancel{5}$ $\cancel{15}$ $\cancel{105}$	$\cancel{742}$ 7
g. w. 105	100 lb. net weight.	$\cancel{5}$ $\cancel{15}$	$\cancel{100}$
lb. 1	9 sh.		$\cancel{9}$ 3
7½	$1 the equality required.	$\cancel{106}$	2
106	100 discount } extra	$\cancel{100}$	$\cancel{100}$ 4
100	120 profit } conditions.		120
			960 Ans.

In the usual rule of arithmetics now in use, the above question would require five statements and operations in the rule of three.

Little Companion published at Philadelphia, copy right secured 16th, August, 1830, and inserted on page 7 of a new system of Arithmetic by *C. Tracy, A. M.* Principal of Norwich Academy, published by Durie & Peck, New Haven.

Bought 742 lb. of wool, a deduction of 5 per cent. from the gross weight being made for dust &c. for the net weight I paid 9 sh. per lb. New York currency, and for ready money was allowed a deduction of 6 per cent, now allowing that I sold the same so as to realise a gain of 20 per cent. how much money did I receive?

$$\frac{\overset{7}{\cancel{742}}\ \ \overset{20}{\cancel{100}}\ \ \overset{3}{\cancel{9}}\ \ \cancel{100}\ \ \overset{15}{\cancel{120}}}{\underset{\underset{\cancel{5}}{\cancel{15}}}{\cancel{105}}\ \ \cancel{8}\ \ \cancel{106}\ \ \cancel{100}}=900 \text{ Ans.}$$

Solution.

$\cancel{15}$ $\cancel{105}$	$\cancel{742}$ $\cancel{20}$ 7
$\cancel{8}$	100
$\cancel{106}$	9
$\cancel{100}$	$\cancel{100}$
	$\cancel{120}$ $\cancel{15}$
	900

To escape the charge of plagiary the ingenious author above named has substituted a horizontal line instead of my perpendicular one, but has not given the student the least idea of statements or the Properties of Figures and Numbers: He directs him to place multipliers above that horizontal line and divisors below it, and then cancel; but he offers no rules for determining multipliers and divisors nor any proper directions for *cancelling;* of his proficiency in which he gives a most suspicious evidence in some places—see p. 133 for instance, where he allows an answer to be stated $1\frac{3}{108}$ instead of $1\frac{1}{36}$.

Thus: 1st. how much net weight is in 742 lbs. gross weight, at a deduction of 5 per cent. tret.

```
105  :  100  ::  742
                 100
                74200(706⅔ Answer.
                735
                 700
                 630
                  70
                 105 = 2/3
```

2d. how many £ sterling for 706⅔ net weight at 9sh.

```
1  :  9  ::  706⅔
               9
            6354
               6
            6360=318 £.   N. Y. Currency
```

which we must bring to dollars.

```
        318
         20
      8)6360
         795 £.
```

Now we have to take the discount off,

```
106  :  100  ::  795
                  100
                79500(750 Ans.
                742
                  530
                  530
```

now adding the 20 per cent.

```
100  :  120  ::  750
                 120
               90000  Ans. $900.
```

A merchant bought in London, 975 ells. at 7sh. 6d. per ell. the cost of commission, transportation and duty

amounted to 40 per cent. exchange 10 per cent. premium ; for how many cents must 1 yard be sold in the United States, to gain $12\frac{1}{2}$ per cent.

Statement.

cts.	1 yd.
yds. 5	4 ells.
1	$7\frac{1}{2}$ sh.
$4\frac{1}{2}$	$1
1	100 cts.
100	110 premium.
100	140 duty &c.
100	$112\frac{1}{2}$ profit and gain.

Solution.

$\cancel{5}$	$\cancel{4}$
$\cancel{2}$	$\cancel{15}$ 3
$\cancel{9}$	$\cancel{2}$
$\cancel{100}$	$\cancel{100}$
$\cancel{100}$	$11\cancel{0}$
$\cancel{25}$ $\cancel{100}$	$\cancel{140}$ 7
$\cancel{2}$	$\cancel{225}$ $\cancel{25}$
	231 cts. $2 31.

This queštion would require in the usual method 7 different statements and solutions.

To prove the correctness of our calculation we may inquire how many dollars must have been paid for 975 ells. on the same condition.

Statement.

$	975 ells.
1	$7\frac{1}{2}$ sh.
$4\frac{1}{2}$	$1
1	100 cts.
100	110 premium.
100	140 duty &c.

Solution.

	$\cancel{975}$ 325
$\cancel{2}$	$\cancel{15}$ 5
$\cancel{3}$ $\cancel{9}$	$\cancel{2}$
	$\cancel{100}$
$\cancel{100}$	$11\cancel{0}$
$\cancel{100}$	$14\cancel{0}$
	2502,50

We may now say, if 975 ells. cost $2502 50 cts. what must 1 yard be sold for in the United States to gain $12\frac{1}{2}$ per cent?

Statement.

cts.	1 yd.
5	4 ells.
975	2502 50 cts.
100	$112\frac{1}{2}$ gain.

Solution.

$\cancel{5}$	$\cancel{4}$ $\cancel{2}$
$\cancel{3}$ $\cancel{39}$ $\cancel{975}$	$\cancel{25025}\cancel{0}$ $\cancel{1001}$ 77
$\cancel{5}$ $\cancel{100}$	
$\cancel{2}$	$\cancel{225}$ $\cancel{9}$ 3
	231 cts. the same answer,

and we have proved the correctness of our first result.

A merchant had $5\frac{8}{9}$ cwt. of sugar at $6\frac{3}{4}$d. per lb. which he bartered for tea at $8\frac{5}{8}$ of a shilling per lb. How many lbs. of tea did he receive for his sugar?

Statement.

lb. tea	$5\frac{8}{9}$ cwt. sugar.
cwt. 1	112 lbs.
lb. 1	$6\frac{3}{4}$ d.
d. 12	1 sh.
sh. $8\frac{5}{8}$	1 lb. tea.

Solution.

$\cancel{9}$	53
	$\cancel{112}$ 56
$\cancel{3}$ $\cancel{4}$	$\cancel{27}$ $\cancel{3}$
$\cancel{4}$ $\cancel{12}$	
69	$\cancel{8}$ $\cancel{2}$
69	2968($43\frac{1}{69}$ lbs. tea.

If a staff 4 feet high cast a shadow 7 feet long, what will be the height of a steeple whose shadow at the same time is 189 feet?

Statement.

real height	189 feet shade
shade 7	4 real height

Solution.

	$\cancel{189}$ 27
$\cancel{7}$	4
	108 feet Ans.

If I buy 20 pieces of cloth, each 20 yards, for $12\frac{1}{2}$ sh. per yard, what is the amount?

Statement.

£	20 pieces
1	20 yards
1	$12\frac{1}{2}$ sh.
20	1 £

Solution.

	$\cancel{2}0$
	$\cancel{20}$
$\cancel{2}$	25
$\cancel{20}$	
	250 £ Answer.

Suppose Amsterdam rates on London 34 s. 3 d. Flemish per £ sterling, and on Lisbon 52 d. Flemish for 400 rees, how then ought we to estimate the exchange between London and Lisbon, that is, how many d. sterling will be equal to 1 milree?

Statement.		Solution.	
d. ster'g.	1 milree	~~400~~	100~~0~~
1	1000 rees	~~12~~	52
400	52 d Fl.		2~~0~~
12	1 sh. Fl.	137	~~12~~
$34\frac{1}{4}$	1 £ ster'g		~~4~~
1	20 sh.	137	10400(75d. 3f. $\frac{89}{137}$ Ans.
1	12 d ster'g		959

810
685
125
4
500
411
89

$\frac{89}{137} = \frac{89}{137}$

A supercargo had sold goods in France for 63000 francs which he wished to send to his employers in America. Exchange direct from France to America being at 5 francs 25 centimes per dollar, from France to England exchange was at 24 francs 80 centimes per £ sterling, and from England to America $8\frac{1}{2}$ per cent. premium, commission 1 per cent. by which way can he send the money to the greatest advantage.

Direct Statement.		Solution.	
$	63000 francs		~~63~~000 3
$5\frac{1}{4}$	$1	~~21~~	4
			12000 Dolls. Ans.

By England.

$	63000 francs
$24\frac{4}{5}$	1 £ st.
9	$40
100	108½ premium
101	100 commis.

Solution.

31 ~~124~~	~~63~~000 7
	~~40~~ ~~20~~ 5
~~2~~	217
~~9~~	~~100~~
~~100~~	5
101	
3131	37975000(12,128$\frac{352}{3131}$

Therefore by the way of England his employers will receive - - - - - - - - - - - - $12,128$\frac{352}{3131}$

But direct - - - - - - - - - - 12,000

Difference in favour of his employers $128.

It will be perceived that by the extended application of the simple *Chain Rule* (as it is called) all arithmetical operations which in other systems are arranged under the heads of *Single Rule of Three*, (direct and inverse) *Practice*, *Interest*, *Discount*, *Loss and Gain*, *Barter and Exchange*; may be easily and with simplicity solved, and even combined; but it may be proper to give a few examples on some of those rules, embraced under Simple Proportion, before we proceed to Compound Proportion.

Barter and Exchange are worked rigidly by the rule given, according to the price of exchange.

Loss and Gain is worked also by the same rule, and can only require three different answers, viz. 1st, what is gained or lost by the whole transaction. 2d, how much per cent. is the gain, and 3d, how must we sell it to gain a fixed per cent.

1st. If I buy 100 yards of goods for $1 25 a yard, and sell it at $1 50 per yard—or if I sell it at $1, this fact never varies, that the difference between the amount of sale and cost is the gain or loss, in either transaction, as

1st. Case No. 1. 100 yards sold at \$150=\$150.
but, 100 yards bought at \$125=\$125.
Gain in the whole. \$25.

Again, No. 2. 100 yards bought at \$125=\$125.
but, 100 yards sold at \$100=\$100.
\$25. Loss in the transaction.

2d. Case No. 3. If we inquire on the same supposition what is the gain or loss *per cent.* by the different prices.

Statement.

per cent. gain	\$100
cost \$125.	\$150 sale.

Solution.

$$\begin{array}{r|l} & \cancel{100}\ 20 \\ \cancel{5}\ \cancel{125} & \cancel{150}\ 6 \\ \hline & 120 \text{ answer.} \end{array}$$

That is for \$100 I have obtained \$120, or gained 20 per cent.

No. 4. Again. If we inquire for the loss per cent. as here:

Statement.

per cent. loss	\$100
\$125	\$100

Solution.

$$\begin{array}{r|l} & \cancel{100}\ 20 \\ \cancel{5}\ \cancel{125} & \cancel{100}\ 4 \\ \hline & \$80 \end{array}$$

wherefore the loss on \$100 is \$20, or 20 per cent.

No. 5. Suppose I purchased some cloth at \$120 and sold it at \$150, what was the gain per cent?

Statement.

per cent. gain	\$100
120	150

Solution.

$$\begin{array}{r|l} & \cancel{100}\ 25 \\ & \cancel{150}\ 5 \\ \cancel{4}\ \cancel{120} & \\ \hline & 125 \text{ answer, or} \end{array}$$

25 per cent.

The amount may also be either Cts. or £ sterling, &c.

No. 6. Purchased some velvet at 18sh. and sold the same, being damaged, for 15sh., what was the loss per cent.

Statement.

$$\begin{array}{r|l} \text{per cent. loss} & 100 \\ 18 & 15 \end{array}$$

Solution.

$$\begin{array}{r|l} & 100 \\ 6\ \cancel{18} & \cancel{15}\ 5 \\ \hline 6 & 500\ (83\tfrac{1}{3} \end{array}$$

therefore $\underline{16\tfrac{2}{3}}$ pr. ct. loss.
100

* Case 3d. If we want to gain a certain per cent.

Suppose Ex. 5 in case 2d.

Statement.

$$\begin{array}{r|l} \text{the selling price} & 120 \text{ buying price.} \\ \text{for } 100 & 125 \text{ avails.} \end{array}$$

Solution.

$$\begin{array}{r|l} & \cancel{120}\ 6 \\ \cancel{2}\ \cancel{100} & \cancel{125}\ 25 \\ \hline & 150 \end{array}$$

150 therefore is the selling price on this condition, and having laid out 120

30 gain in the whole, and as

$$\begin{array}{rlll} & 100 & \text{has gained} & 25 \\ & \underline{20} & \text{will gain} & \underline{5} \\ \text{laid out} & 120 & \text{on which} & 30 \text{ is gained.} \end{array}$$

Suppose we have bought some goods at $96 and sold them for $112, what is the gain per cent. by the rule given?

Statement.

$$\begin{array}{r|l} \text{gain per cent.} & 100 \\ \text{cost. } 96 & 112 \text{ avails.} \end{array}$$

Solution.

$$\begin{array}{r|l} & \cancel{100}\ 25 \\ 3\ \cancel{12}\ \cancel{96} & \cancel{112}\ 14 \\ \hline 3 & 350 = 116\tfrac{2}{3} \end{array}$$

or $16\tfrac{2}{3}$ per cent.

* By this statement, which I have always preferred, we obtain the per cent. above 100, and have not to make a subtraction from buying and selling prices, which often may be tedious.

We see here in an instant that 16$\frac{2}{3}$ is the gain per cent.; but if we wish to have the answer of 16$\frac{2}{3}$ per cent. merely by the calculation, we are at first obliged to seek for the difference of the buying and selling price, as here,

sold for	112
bought for	96
difference	16.

Now we would have the following statement:

Statement.

$$\begin{array}{r|l} \text{gain per cent.} & 100 \\ \text{cost. } 96 & 16 \text{ gain.} \end{array}$$

Solution.

$$\begin{array}{r|l} & \cancel{1}\cancel{0}\cancel{0} \ 50 \\ 3 \ \cancel{6} \ \cancel{9}\cancel{6} & \cancel{1}\cancel{6} \\ \hline 3 & 50=16\tfrac{2}{3} \text{ per cent.} \end{array}$$

By the following question it will appear that the last method would be very tedious in some cases.

Suppose I have bought some goods at 73$\frac{3}{4}$d. per yard and sold them at 82$\frac{4}{5}$ per yard, what was the gain per cent?

Statement.

$$\begin{array}{r|l} \text{gain} & 100 \\ 73\tfrac{3}{4} & 82\tfrac{4}{5} \end{array}$$

Solution.

$$\begin{array}{r|l} & \cancel{1}\cancel{0}\cancel{0} \ \cancel{2}\cancel{0} \ 4 \\ 59 \ \cancel{2}\cancel{9}\cancel{5} & 4 \\ \cancel{5} & 414 \\ \hline 59 & 6624(112\tfrac{16}{59} \end{array}$$

By the other method we would be obliged to take 73$\frac{3}{4}$ from 82$\frac{4}{5}$ thus

$$\begin{array}{r} 82\tfrac{4}{5} \\ 73\tfrac{3}{4} \\ \hline 9\tfrac{1}{20} \end{array}$$

difference; now we have

Statement.

$$\begin{array}{r|l} \text{gain} & 100 \\ 73\tfrac{3}{4} & 9\tfrac{1}{20} \end{array}$$

Solution.

$$\begin{array}{r|l} 59 \ \cancel{2}\cancel{9}\cancel{5} & \cancel{1}\cancel{0}\cancel{0} \ \cancel{5} \\ & 4 \\ \cancel{2}\cancel{0} & 181 \\ \hline 59 & 724(12\tfrac{16}{59} \text{ answer.} \end{array}$$

The rule of *Discount and Amount* is similar to *Interest*, which belongs to compound proportion, but is sometimes

reduced to simple proportion by compounding the time and the rate per cent. together, which product is added to 100 either discount or amount as the case may require—as,

$$\begin{array}{c} \text{Discount.} \\ \hline 100 \end{array} \qquad\qquad \begin{array}{c} \text{Amount.} \\ \hline 100 \end{array}$$

Supposing we had to calculate the present worth of a certain sum for 3 yrs. at 6 per cent. $3 \times 6 = 18$, we would add this 18 to 100 and have the proportion $118 = 100$.— But if we would require the amount, we would add 18 to 100 of amount and have $100 = 118$.

No. 1. Required the present worth of \$590, due in 3 years at 6 per cent?

Statement.

$$\begin{array}{r|l} \text{present worth} & \$590 \\ 118 & 100 \end{array}$$

Solution.

$$\begin{array}{r|l} & \cancel{590}\ 5 \\ \cancel{118} & 100 \\ \cline{2-2} & 500 \text{ Answer.} \end{array}$$

Subtracting 500 from $590 = 90$ the discount. But if we want to have the discount at once by the calculation, we have the proportion $118 = 18$ thus,

Statement.

$$\begin{array}{r|l} \text{discount} & \$590 \\ 118 & 18 \end{array}$$

Solution.

$$\begin{array}{r|l} & \cancel{590}\ 5 \\ \cancel{118} & 18 \\ \cline{2-2} & 90 \text{ discount.} \end{array}$$

The rule of Discount is applied by all merchants, brokers and bankers in exchange and commission, or in general when a sum of money is paid before it becomes due; for instance, suppose the exchange between Philadelphia and New York is at 6 per cent. premium in funds of N. York, that is, \$100 in New York are equal to \$106 in Philada., or \$106 in Philadelphia are equal to \$100 in New York. Suppose a New Yorker had to pay in Philadelphia \$10600, he wants to discharge that sum, he applies to a broker,

banker or merchant, for a check or bill of exchange; how much must he pay for it to discharge his debt in Philada.

Statement.			Solution.	
of N. Y. $	10600	of Phila.		~~1~~0~~6~~00
Phila. 106	100	of N. Y.	~~106~~	100
				10000 Answer.

It will be seen here plainly that if 6 per cent. interest had been deducted from $10600, the interest would be $686; the $10000 to be paid is called the nett proceeds, from which 6 per cent. interest is equal to $600. Suppose now you have sent to a commission merchant $10600 worth of goods at 6 per cent. commission; here it would appear to any person not acquainted with commercial accounts that the commission merchant had a right to take 6 per cent. interest from the gross amount of $10 600, which would be $636 commission, to be deducted from $10 600, and would leave $99648; but this is not so, he has only a right to take 6 per cent. interest from the nett proceeds or $10000, called in the rule of Discount the present worth. All this is well known by the expert Merchants, Brokers and Bankers, but not generally known by teachers, and is not explained in the text books adopted and used in the schools of the Union.

What will be the amount of a bond of $962 at 6 per cent., of which 4 years interest are in arrears. We compute here 6 per cent. with 4 years=24 and have 100=124, then

Statement.

Amount $	$962
100	124
100	3848
	11544
	1192,88 Answer=$1192 88 cts. Amount

or capital and interest.

Tare and Tret is not properly a rule of Arithmetic. Tare means an allowance or deduction of weight, for boxes, barrels, bags, &c. in which the goods sold or purchased are contained. Tret is a certain allowance for waste in transportation, &c. and if these allowances are deducted the nett amount remains, and is calculated according to conditions. Example.

Bought 84 cwt. of sugar; what is the nett weight if 20lb. per cwt. are allowed for Tare? and what is the sum to be paid for it at 7 cents per lb.

cwt. lbs. tret.	gr. wt.
84 × 20 = 1680	84 × 112 = 9408
	1680 tret.
	7728 nett weight.
	7 cts.
	540 96 Answer.

Practice is no rule in Arithmetic: in other systems it is said to be a contraction of the rule of three or proportion, and is worked by the assistance of aliquot parts; its practical use is to shorten operations; but Practice as regarded in this system has no boundaries, and all operations of calculation, by changing factors to easy multipliers and divisors by means of the *properties of figures and numbers*, may be called Practice.

Example 1. If 1 cwt. cost 4£. 17sh. 4d., what will 9cwt. 3qrs. 14lb. cost?

Usually	4	17	4
			9
	43	16	"
$\frac{1}{2}$	2·	8	8
$\frac{1}{4}$	1	4	4
$\frac{1}{8}$		12	2
	48	1	2

Reasoning differently, thus
9 cwt. 3qrs, 14lb. is = to
10cwt. $-\frac{1}{8}$; we have s. d.
4 17 4 × 10 $-\frac{1}{8}$ = 97 4 = 12 2.

		10	
48	13	4	
	12	2	deduct.
48	1	2·	

Ex. 2d. 1cwt. cost \$8 64 cts. what will 9cwt. 3qr. 14lb. cost?

Usually \$8.	64 cts.		New system.		
			\$8	64 cts.	10cwt.—$\frac{1}{8}$.
	9			10	
	77	76	86	40	
$\frac{1}{2}$	4	32	1	08	deduct $\frac{1}{8}$.
$\frac{1}{4}$	2	16	85	32	result the same.
$\frac{1}{8}$	1	08			
	85	32			

Example 3d. If $4\frac{1}{2}$ pieces of Broad cloth have been bought for 486£. 12sh. 9d. what must be paid for 23 pieces.

Statement.

£.	23 pieces
$4\frac{1}{2}$	486£. 12sh. 9d.

Solution.

	$\cancel{2}\cancel{2}\frac{1}{2}+\frac{1}{2}=23$		
	5		
$\cancel{4}\frac{1}{2}$	486£.	12sh.	9d.
	2433	3	9
	54	1	5
	2487	5	2.

Illustration, changing $23=22\frac{1}{2}+\frac{1}{2}$ and as $4\frac{1}{2}$ in $22\frac{1}{2}=5$ we have to multiply 486£. 12sh. 9d. by 5=2433£. 3sh. 9d. now we have to multiply 486£. 12sh. 9d. by $\frac{\frac{1}{2}}{4\frac{1}{2}}$ reduced to a simple fraction$=\frac{1}{9}$, wherefore, dividing by 9=54. 1. 5. added to 2433. 3. 9=2487£. 5sh. 2d.

It has been already said that we are not obliged to work the rules of geometrical proportion with fractions or compound numbers, if we wish not to do so, because we are enabled to reduce them to whole numbers: the above questions may be stated and worked with whole numbers, thus:

Statement.

£.	23 pieces.
$4\frac{1}{2}$	$486\frac{5}{8}\frac{1}{80}$£.

Solution.

		23
3	$\cancel{9}$	$\cancel{2}$
40	$\cancel{8}\cancel{0}$	$\cancel{3}\cancel{8}\cancel{9}\cancel{3}\cancel{1}$ 12977
120		38931
		25954
Result the same as above.		298471(2487£. 5sh. 2d.

DUODECIMALS

Are calculation by twelfths as Decimals are by tenths: they are worked, by those who are not acquainted with fractions, like compound numbers. It is the rule by which we ascertain solid or superficial measure: the unit of a foot is divided into 12 inches, and the inch into 12 seconds, or 12th parts of an inch, &c. for instance:

Ex. 1st. It is required to floor a room 33 feet 4 inches long, and 18 feet 9 inches broad; how many square feet of board are required to do this?

Worked by duodecimals.

ft.	in.	
33.	4.	
18.	9.	
264		
33		
6		
24	9	
	3	
625	"	square feet.

Worked by fractions.

$\cancel{3}$	$\cancel{1}\cancel{0}\cancel{0}$	25
$\cancel{4}$	$\cancel{7}\cancel{5}$	25
	625	square feet.

Illustration. In working with duodecimals we must observe that feet multiplied by feet produce feet, but that feet multiplied by inches produce inches, and that inches by inches produce seconds, &c. In the above question we compute first, 33 feet by 18 feet=594 ft. 0 in.,

compute first, 33 feet by 18 feet=	594 ft.	0 in.,
then 2d. 18 feet by 4 in.=72 in.÷12=	6	
3d. 33 ft. by 9 in.=297 in.÷12=	24	9
4th. 4 in. by 9 in.=36÷12=		3
	625 feet.	

The freight in vessels from one port to another is generally calculated by the space of cubic feet which merchandise will occupy, and as here three sides must be multiplied together to find the contents, it will be found very tedious

to work such questions by duodecimals, whereas by the simple fractional operation we will not find any difficulty at all.

Example 2d. Shipped to London a bale of cotton 7 feet 6 inches long, 5 feet 4 inches broad, and 3 feet 9 inches thick; what is the contents, and how much freight must be paid at 4 cts. per cubic foot.

Worked by duodecimals.

ft.	in.	
7	6	
5	4	
35		
4	10	
	2	
40	"	"
3	9	
120		
30		
150	feet contents.	
at 4	cts.	
6.00	Result $6.	

Worked by fractions,

~~2~~	15
~~3~~	~~10~~ ~~4~~ 2
~~4~~	~~15~~ 5
	150 feet.
	4 cts.
	6.00 Result $6.

We need not indicate which of these means of solution is the most desirable.

COMPOUND PROPORTION

Teaches from a known relation between three or more numbers, to find a new relation between other numbers consistent with the first supposition.

Rule. We read the question, and as soon as the *name* of the answer required appears, place the sign or mark of this name on the *left* side of the line, and all the conditions

whereupon the question more immediately lies on the *right* hand side of said line; we then go back to the remaining terms of the question, and place them on the left hand side in the same order as those on the right hand side, except that of the same *name* of the *answer*, which must always stand on the right side of the line, last of all. We then proceed to contract and cancel as already shown.

1st. If 4 men in 5 days eat 7 lbs. of bread, how many lbs. will 16 men eat in 15 days?

Statement.

lbs. bread	16 men.
men 4	15 days.
days 5	7 lbs. bread.

Solution.

$$\begin{array}{r|l} & \cancel{16}\ 4 \\ \cancel{4} & \cancel{15}\ 3 \\ \cancel{5} & 7 \\ \hline & 84 \text{ lbs. Answer.} \end{array}$$

2d. If 10 bushels of oats be sufficient for 18 horses 20 days, how many bushels will serve 60 horses 36 days?

Statement.

bushel	60 horses.
horses 18	36 days.
days 20	10 bushels.

Solution.

$$\begin{array}{r|l} & 60 \\ \cancel{18} & \cancel{36}\ \cancel{18} \\ \cancel{20} & 1\cancel{0} \\ \hline & 60 \text{ Answer.} \end{array}$$

3d. If $350 in half a year gain $10 50 cts. interest, what will be the interest of $400 for 4 years?

Statement.

cts. interest	$400.
$350	4 years.
year ½	$10 50 cts.

Solution.

$$\begin{array}{r|l} \cancel{7}\ \cancel{350} & \cancel{400}\ 8 \\ & 4 \\ 1 & 2 \\ & \cancel{10}\ \cancel{50}\ 150 \\ \hline & 9600 \text{ cts.} = \$96. \end{array}$$

Before we proceed any further it is necessary to say a word on *Direct* and *Inverse Proportion.* Inverse Proportion cannot exist in the *single rule of three.* The introduction of questions involving this peculiarity into simple

proportion, has confused our best teachers and disfigured our arithmetics, as it absolutely requires a compound proportion with five terms at least, to propose an inverse proportion; those arithmetics which have stated inverse questions in simple proportion, have suppressed the two passive terms, (which we call the *effect*.) For instance: If 12 men can build a house in 48 days, in what time could 36 men build it?

In reading this question it would seem to belong to the single rule of three, because only three terms are distinctly given, but the two other terms (which are the passive ones, or the effect) have been suppressed or passed over; the five terms required for the formation of an inverse proportion will be plainly seen in the above question when candidly stated.

If 12 men in 48 days can build 1 *house*, how many days will it require 36 men to build 1 *house?*

Here we see that the effect (viz. the house) produced by the specified causes, has been omitted.

I borrowed of my neighbour $1200 for 6 months, and I promised to do him the like kindness; some time after I handed him $ 900, how long may he keep this sum to requite himself?

We might readily believe, in reading this question carelessly, that it belonged to simple proportion; because it presents only three terms! but here the *effect*, which is the *interest* (6 or 7 per cent.) has been omitted, and so it is in all inverse examples presented under the single rule of three. No *inverse* question can be fairly stated by three terms.

Direct Proportion is where the answer is an *effect*.

Inverse Proportion is where the answer is a *cause*.*

* See preface to this edition.—All the terms *acting*, *producing* or *consuming* are CAUSES, viz. *men*, *horses*, *time*, *capital*, *length*, *breadth*, *thickness*, or *parts of a compound*, *&c.* EFFECTS are the result or consequence of said causes, viz. *work*, *wages*, *interest*, *superficial and solid contents*, &c. By fixing these distinctions in the memory, the student will soon be able to apply the criterion with ease and certainty.

4th. If 7 men reap 84 acres of wheat in 12 days, how many *men* will reap 100 acres in 5 days?

Statement.

men	100 acres.		~~100~~	20 men.
84	(5) days inverse.	~~7~~ ~~84~~	~~12~~	
inverse (12)	7 men.	~~5~~	~~7~~	

We here observe that the answer will be a *cause*, (the men and the days produce the reaping,) the proportion is therefore *Inverse;* we make our statement as in Direct Proportion, and transpose or *invert* all the given *producing* terms, except that which is of the same name as the answer.* We then proceed to cancel and contract, and find our answer as previously directed.

5th. If 4 compositors in 16 days of 12 hours long, can compose a work of 14 sheets of 24 pages in each sheet, 44 lines in a page, and 40 letters in a line: In how many days of 10 hours long, may 9 compositors compose a volume to be printed on the same letter consisting of 30 sheets, 16 pages in a sheet, 48 lines in a page, and 45 letters in a line.

Statement.

	days	10 hours.	inv.
inv.	hours 12	9 comp.	inv.
inv.	comp. 4	30 sheets.	
	sheets 14	16 pages.	
	pages 24	48 lines.	
	lines 44	45 letters.	
	letters 40	16 days.	

Solution.

1~~0~~	12
~~9~~	~~4~~
7 ~~14~~	3~~0~~
~~24~~	~~16~~ ~~2~~
11 ~~44~~	~~48~~ 2
~~8~~ ~~40~~	~~45~~ ~~5~~
	16
77	$1152 = 14\frac{74}{77}$ Ans.

Considering here that days, a term of the cause, is required, this question is inverse; and as hours and compositors are terms of causes, they are changed in the solution, and then reduced by cancellation, &c.

* The *effect* terms, and the *cause* term which is of the same *name* as the answer, never change their *places*.

If, as generally in practice, the statement and solution are performed at once, we enclose those inverse terms in parentheses that we may not be confused in cancelling, and place them in their respective places below, as

Statement and solution.

days	(10) hours.
hours (12)	(9) comp.
comp. (4)	$3\cancel{0}$ sheets.
sheets 7 $\cancel{14}$	$\cancel{16}$ $\cancel{4}$ pages.
pages $\cancel{24}$	$\cancel{48}$ $\cancel{2}$ lines.
lines 11 $\cancel{44}$	$\cancel{45}$ $\cancel{5}$ letters.
letters $\cancel{40}$	16 days.
$\cancel{2}$ $\cancel{10}$	12
$\cancel{5}$	$\cancel{4}$ 2
77	$1152 = 14\frac{74}{77}$ Answer.

No. 6. If a bar of iron 4 feet long, 3 inches broad, and $1\frac{1}{2}$ inch thick weighs 36 lbs., what will a bar weigh that is 6 feet long, 4 inches broad, and 2 inches thick?

Statement.

lb.	6 feet long.
ft. 4	4 in. broad.
inch 3	2 inches thick.
inch $1\frac{1}{2}$	36 lb.

Solution.

$\cancel{4}$	$\cancel{6}$ 2
$\cancel{3}$	$\cancel{4}$
	2
	$\cancel{36}$ 12
$\cancel{3}$	2.
	96 lbs.

We perceive here that the cube of the bar of iron is required, or its representative in pounds, it is therefore a direct question.

No. 7. If a garrison of 600 men have provision for 5 weeks, allowing each man 12 ounces a day, how many men may be maintained 10 weeks with the same provisions if each man is limited to 8 ounces a day?

Statement.		Solution.	
men	10 weeks. } in.	$\cancel{2}\ \cancel{10}$	$\cancel{5}$
{ weeks 5	8 ounces. }	$\cancel{8}$	$\cancel{12}$ 6
{ ounces 12	1 provision.		$\cancel{600}$ 75
provision 1	600 men.		450 men Ans.

No. 8. A wall is to be built 700 yards long in 29 days, after 12 men had been employed on it for 11 days, it was found that they had built only 220 yards; how many more men must be put on to finish it in the given time?

taking from 700 yards and 29 days
220 " " 11

Remains to be built 480 yards in 18 days: therefore

Statement.		Solution.	
men	480 yds.	$\cancel{2}\ \cancel{220}$	$\cancel{480}$ 16
yds. 220	18 days inverse.	$\cancel{3}\ \cancel{18}$	$\cancel{11}$
inv. day 11	12 men.		$\cancel{12}\ \cancel{6}$
			16 men Ans.

therefore 4 men more.

Teachers will perceive that in *Compound Proportion* we need not give a great number of original examples, since every such example contains as many *proof* questions or new examples as there are given terms in the first question; by classing terms of similar name in pairs, the proof questions may be easily found: viz.

If 5 men can make 300 pairs of boots in 40 days, how many men must be employed to make 900 pairs in 60 days.

Statement.		Solution.	
men	900 pairs	$\cancel{300}$	$\cancel{900}\ \cancel{3}$
pairs 300	60 days inverse	$\cancel{2}\ \cancel{60}$	$\cancel{40}$ 2
inverse, days 40	5 men.		5
			10 men Ans.

Classing now the 6 terms into pairs, thus:

men.	days.	boots.
5	40	300
10	60	900

From the given five terms we have found the 6th term, (10 men) in the lower line. If we now omit one term in the upper line, we have the supposition on the terms of the lower line: Suppose we drop the term 40 days in the upper line, then we have the question—

If 10 men in 60 days can make 900 pairs of boots, in how many days can 5 men make 300 pairs.

Statement.

days	5 men, inverse.
inverse, men 10	300 pairs.
pairs 900	60 days.

Solution.

~~5~~	~~10~~ 2
~~3~~ ~~900~~	~~300~~
	~~60~~ 20
	40 days the term.

Proof of our first answer.

Suppose we now drop the term 900 in the lower line, then we will have the following question.

If 5 men in 40 days can make 300 pairs of boots, how many pairs can 10 men make in 60 days?

Statement.

pairs	10 men.
men 5	60 days.
days 40	300 pairs.

Solution.

~~5~~	1~~0~~
~~40~~	~~60~~ ~~15~~ 3
	300
	900

NOTE.—Here the *effect* being required, the proof question is *direct*.

Finally it may be observed that questions in Inverse Proportion can be solved by two or more statements in Simple Proportion, according to the number of Simple Proportions which may have been compounded together.

Suppose the question, if 7 men can reap 84 acres of wheat in 12 days, how many men can reap 100 acres in 5 days?

If we say here, how many acres in 12 days, if 5 days= 100 acres.

Statement.		Solution.	
acres	12 days.		12
days 5	100 acres.	$\cancel{5}$	$\cancel{100}$ 20
			240 acres.

Now we would have, if 84 acres=7 men, how many men=240 acres?

Statement.		Solution.	
men	240 acres.		$\cancel{240}$ 20 men Ans.
acres 84	7 men.	$\cancel{12}$ $\cancel{84}$	$\cancel{7}$

This shows plainly that Simple Proportional questions cannot be inverse.

If 50£ in 5 months gain $2\frac{37}{144}$£ interest, in how many months will $13\frac{1}{3}$£ gain $1\frac{1}{12}$£ interest?

Statement.		Solution.	
months	$13\frac{1}{3}$£ inverse.	$\cancel{40}$	3
inverse £50	$1\frac{1}{12}$ interest.	$\cancel{12}$	$\cancel{13}$
gain $2\frac{37}{144}$	5 months.		$\cancel{5}$
			$\cancel{50}$
		$\cancel{13}$ $\cancel{65}$ $\cancel{325}$	$\cancel{144}$ $\cancel{12}$ 3
			9 months Ans.

If \$$333\frac{1}{3}$ gain \$15 interest in 9 months, what sum will gain \$6 interest in 12 months.

Statement.		Solution.	
sum	6 interest.	$\cancel{2}$ $\cancel{12}$	$\cancel{6}$
interest 15	12 month inv.	$\cancel{5}$ $\cancel{15}$	$\cancel{9}$ $\cancel{3}$
inverse, month 9	$333\frac{1}{3}$ sum.	$\cancel{3}$	$100\cancel{0}$
			100 Ans.

Bank discount is different from the rule of discount in commercial business; it is calculated like interest, and taken off from the sum advanced.

What is the discount of $6000 for 2 months at 6 per cent. with the three days' grace? The Banker calculates interest for 63 days.

Statement.

interest	6000
100	63
360	6

Solution.

	$\cancel{6000}$
$1\cancel{00}$	63
$\cancel{3}\cancel{6}\cancel{0}$	$\cancel{6}$*
	$63 interest

which he takes from 6000, and pays you $5937; whereas if you had borrowed on mortgage $6000 from a private person you would have paid only $60 interest at 6 per cent. after the expiration of two months; wherefore we must not take *Bank discount* either for interest or for discount, where money is paid before it comes due.

PROPORTIONAL DIVISION OF LOSS & GAIN

Comprises *Fellowship*, *Assessment of Taxes*, *Bankruptcy*, *General average*, *Bank Dividends*, and a great many useful and intricate questions.

Rule.—Add the different stocks or proportional parts; place this sum as a denominator under the loss or gain, and by this fraction multiply each man's stock for each man's proportional share or answer. By Compound Fellowship the time, (days, months, or years) is multiplied by the amount each man put in, and these respective products form their several stocks, then add and proceed as above directed.

To shorten the operation, use *Cancellation* and *Contraction* in the stocks as well as in the fractions.

Ex. 1st. A, B & C traded together and gained $120;

* Here we have multiplied 6 by 6 and cancelled the product against 36.

A's stock was \$140, B's stock \$300 and C's \$160, what was each man's share of the gain?

A's	Stock	140		
B's	"	300	whole gain	120
C's	"	160	whole stock	$\overline{600}=\frac{1}{5}$.
		$\overline{600}$		

Then $140\div5=28$ A's gain
$300\div5=60$ B's "
$160\div5=32$ C's "

Having placed the gain, 120 as a numerator, and added the stocks=600 as denominator, the fraction thus obtained is the multiplier of each man's stock for each man's share of the gain, but having reduced this fraction to $\frac{1}{5}$, we have only to multiply by $\frac{1}{5}$, or to divide each man's stock by 5 for each man's share.

We will take the same question and contract the stocks in the same proportion.

	Reduced.		
A	$140\div20=\ 7=28$	gain	$\frac{120}{30}=\frac{4}{1}$.
B	$300\div20=15=60$	stock	
C	$160\div20=\ 8=32$		
	$\overline{30}\ \ \overline{120}$ total gain as before.		

Having reduced the stocks to 7, 15, 8=30 (by contracting them by the factor 20) placing 30 under the gain 120, reducing it to an improper fraction=$\frac{4}{1}$ we have to multiply the reduced stocks by this fraction, for each man's share as above.

2d. *On Bankruptcy.* A merchant has failed in trade, who owes to A \$604, to B \$728, to C \$368, to D \$120, to E \$76, F \$392, and G 472, but his property is only worth \$696, what is each man's share?

Debts.		Assets
A	604	151
B	728	182
C	368	92
D	120	30
E	76	19
F	392	98
G	472	118
	2760	690

$\frac{696}{2760}=\frac{690}{2760}\frac{1}{4}+\frac{6}{2760}$

To have an easy number to operate with, we omit \$6, and cancel the fraction left $\frac{690}{2760}$=to $\frac{1}{4}$, now dividing each man's demand by 4 we have each man's share for the sum of 690; but as \$6 are yet to be divided in the same proportion, and as 690 stands in proportion with 2760 we have $\frac{6}{690}=\frac{1}{115}$.

Now adding two ciphers to each man's share and dividing by 115, we find the proportion of cents due each creditor.

```
115)15100(1.31 35/115
    115
    ---
     360
     345
     ---
      150
      115
      ---
       35
```

It is hot necessary for us to go through the calculation of every man's share, in cents, (let this be done by the scholar for exercise;) but it is evident, if we multiply the total sum of 690 by $\frac{1}{115}$ that \$6 will be the result.

3d. *Assessing Taxes.* A tax of \$679 is to be assessed on a town or parish of 60 polls; \$75 of the sum to be a poll tax, leaving \$604 for the real and personal property, which by the inventory is found to be \$60,000; how much must a person pay whose real estate in the inventory is \$2000 and \$300 personal property, and who pays for two polls?

Tax.		Polls.		per Poll.
75	÷	60	=	\$1.25

$\frac{604}{60.000}=\frac{1}{100}+4$

Real Estate=	\$2000
Personal Property	\$300
	\$2300
at $\frac{1}{100}$ part	\$23 00
tax of 2 polls 1 25	2 50
	\$25 50 Total tax.

As tax paying communities may be considered permanent associations, not likely to be removed or seriously changed from year to year, it is usual for the officers to establish a balancing fund, by which fractions of excess or deficit can be settled without the difficulty of following out the exact assessment. Such *generalizing* cannot of course be allowed in *Bankruptcies*, but even under these circumstances our calculation may be shortened as in example 2d.

4th. *On General Average.* A vessel on her voyage sustained so much damage that she was obliged to put into port to repair; all expenses amounted to £149 18sh. 9d, these expenses were to be proportionally divided by a general average, thus:

£. sh. 9d.

$$\frac{149\ 18\ 9}{45000} = \text{to}\ \frac{150}{45000} = \frac{1}{300} - 1\text{sh. } 3\text{d.}$$

		value.		loss.
Vessel		6700£	÷300=	$22\frac{2}{3}$
Nett weight		500	"	$1\frac{2}{3}$
Cargo shipped by	A	1200	"	4
" "	B	7800	"	26
" "	C	21500	"	$71\frac{2}{3}$
" "	D	3300	"	11
" "	E	4000	"	$13\frac{1}{3}$
		45000		150£

We may see that it would make a very tedious and long calculation, if these 149£. 18sh. 9d. were to be divided as laid down in some arithmetics; having made the sum easy with a difference 15d. should it be required to divide them exactly, we have less $\frac{15}{150}$=to$\frac{1}{10}$ of a penny less, which would be 15d. on 150£. calculating this on B's share 26 $=\frac{26}{10}$—2d. $\frac{6}{10}=\frac{3}{5}$ leaves for B 25£. 19. $9\frac{2}{5}$d. &c. by exact calculation.

Compound Fellowship, or Partnership.

Rule.—Multiply each man's amount put in, by the time, and their respective products form their several stocks, then proceed as above directed.

5th. A, B and C, traded together; A put in 70£. for 10 months, B 180£. for 5 months, and C 200 for 3 months, they gained 132£. what was each man's share?

C	$200\times 3=600=$	$6=36$		$\frac{132}{22} \big| \frac{6}{1}$
B	$180\times 5=900=$	$9=54$		
A	$70\times 10=700=$	$7=42$		
	2200	22	132£.	

6th. A, B, C and D, traded together on a capital of $6000, A put in $\frac{1}{2}$, B $\frac{1}{4}$, C $\frac{1}{6}$, D $\frac{1}{12}$; at the end of 4 years they had gained $2364, what was each man's share of the gain?

		gain.		
A	$\frac{1}{2}\times 6=6=$	1182	(12 multiple.	
B	$\frac{1}{4}\times 3=3=$	591		whole gain.
C	$\frac{1}{6}\times 2=2=$	394		$\frac{2364}{12}=\frac{197}{1}$
D	$\frac{1}{12}\times 1=1=$	197		
	12	2364	total gain.	
	12			

If in fellowship the stocks are expressed in fractions, we find their numerators by the rule of addition of fractions; these several numerators form their respective proportional stocks, after which the sum of these numerators are placed under the loss, or gain, and by this fraction multiplied for the answer.

7th. Suppose the above named gentlemen had agreed to form a stock of $6000, and to put in the above parts; but D died and his part was not put in, therefore

A $\frac{1}{2} \times 6 = \$1289\frac{5}{11}$ (12 multiple. $\frac{2364 \times 6}{11} = \frac{14184}{11}(1289\frac{5}{11}$
B $\frac{1}{4} \times 3 = \quad 644\frac{8}{11}$
C $\frac{1}{6} \times 2 = \quad 429\frac{9}{11}$ for A, and so on for B and C.
$\overline{11} \quad \overline{\$2364}$
$\overline{12}$

It is evident if D had put in $\frac{1}{12}$ of 6000=500, the stock would have been the same as fixed above; but as his share of the stock was 500, the whole stock of A, B and C, was only $5500, and according to *this stock* they had to divide the gain; and not by the part of $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{6}$ as at first might appear.

8th. A, B and C, formed a joint stock of $13176, by which they gained $936. A's money is in trade 4 months, B's 5 months, C's 13 months; their parts of the gain are in the proportion of 2, 3, 4; required the gain and stocks of each partner.

	Prop.	Gain of each.
A	2 =	208
B	3 =	312
C	4 =	416
	9	936 total gain.

$\frac{936 \text{ gain } 104}{9 \qquad 1}$

Now to find the proportion of each man's stock we have to divide their gain by their time, as

months $\frac{208}{4} = 52$ mo. $\frac{312}{5} = 62\frac{2}{5}$ mo. $\frac{416}{13} = 32 = 146\frac{2}{5}$

Reduced to whole nunbers $\frac{13176 | 18}{732 | 10}$

A	52	=	260	× 18	=	4680
B	$62\frac{2}{5}$	=	312	× "	=	5616
C	32	=	160	× "	=	2880
prop. stock,			732			13176 whole stock.

K

Proof by Fellowship with time.

A	\$4680	4 months	18720	÷90=	208
B	5616	5 "	28080	"	312
C	2880	13 "	37440	"	416
			84240		936 total gain.

9th. A, B, and C, would divide \$80 among them in such a manner that B may have \$5 more than A, and C \$10 more than B, required the share of each.

A	1			=	20	80
B	1	+	5	=	25	20
C	1	+	15	=	35	60=20
	3		20		80	

According to the conditions of the question it is plain that A shall have a share, B one and \$5 more, and C one and \$10 more than B, or \$15 more than A; by the principles of increase and decrease, we have then only to subtract 5+15=20 from 80=60, which divided into 3 parts gives 20 for A, for B 20+5=25, for C 20+15=35; here it will be seen that the learner can judge of his process, whereas if he proceed by the rule of *Double Position*, so called, he cannot give a reason for his operations until he has the answer, and is astonished that it corresponds with the conditions of the question. But nothing can be more plain than that if two persons have to divide 12 apples, of which B shall have 2 more than A, let B take 2 off and they have to divide 10; of which A will have 5, and as B has already 2, he will have 7.

10. The account of a school is as follows, viz. $\frac{1}{16}$ of the boys learn geometry, $\frac{3}{8}$ grammar, $\frac{3}{10}$ arithmetic, $\frac{3}{20}$ writing and 9 reading, what number is there of each?

Geometry	$\frac{1}{16}$	5= 5	(80 multiple.
Grammar	$\frac{3}{8}$	10=30	
Arithmetic	$\frac{3}{10}$	8=24	
Writing	$\frac{3}{20}$	4=12	
Reading	9	$\frac{71}{80}+\frac{9}{80}=\frac{80}{80}=$ the whole school.	

The numerators to the fractions give the answer, as

5	learn	geometry,
30	"	grammar,
24	"	arithmetic,
12	"	writing,
9	"	reading.

Having added the fractions, the whole numbers given will represent the difference between the numerator and denominator of said sum; and where we make the numerator and denominator of a fraction balance, it becomes a *perfect quantity*. The whole number given in this example is 9, and if that is $\frac{9}{80}$ it is plain that $\frac{80}{80}$ is the amount of the scholars.—Again,

11. In an orchard $\frac{1}{2}$ of the trees bear apples, $\frac{1}{4}$ pears, $\frac{1}{6}$ plums, 60 of them peaches and 40 cherries: how many trees are in the orchard?

apples $\frac{1}{2}$ 6= 6 (12 multiple.
pears $\frac{1}{4}$ 3= 3
plums $\frac{1}{6}$ 2= 2
peaches 60. $\overline{11}$
cherries 40. $\frac{11}{12}+\frac{1}{12}=\frac{12}{12}$ it is evident that $\frac{1}{12}$ is equal to 100, the whole numbers given, and that 1200 trees are contained in the orchard.

ALLIGATION

Treats of *Mixtures*; showing how to form a compound of a certain general character, from simples each distinct and peculiar: it has been distinguished as *Medial* and *Alternate*.

Medial Alligation gives the amount and characteristics of the simples to determine the character of the compound.

Rule. Multiply the given quantities to be mixed together by their respective prices, after which add their products; add also the given quantities, and by this sum divide the sum of the several products, and the quotient or

result gives the mean or medium price of the mixture per gallon, bushel, hogshead, &c. &c.

Ex. 1. If we take rye at 48 cts. a bushel, barley at 26 cts., and oats at 24 cts., what will the mixture be worth per bushel?

simples.		price.	product.
1	at	48 =	48
1	at	36 =	36
1	at	24 =	24
3			108=36 cts. Answer.

Ex. 2d. If 4 ounces of silver be worth 75 cts. per ounce, and be melted with 8 ounces worth 60 cts. per ounce, what will be the worth of this composition?

4	at 75=	300
8	at 60=	480
12		780=65 cts. Answer.

Alternate Alligation is the reverse of the preceding, and gives the character of the compound whereby to determine that of the simples.

Rule.—The price of the mixture being given, and the prices also of the ingredients, we take one part of each and place opposite its respective price, then add up these two columns, and divide the sum of the prices by the sum of the simples, if the quotient shows the required price, our calculation is complete, but if it does not, we multiply the required price or *mean rate*, by the sum of the simples just named, and find the difference between this product and the sum of the prices already found, if said *product* be greater than said *sum*, we have supposed too little, and must subtract the mean rate from the higher price, and divide our first difference by this last, and the quotient will show how many additional parts of the higher price are required; but if the above named product be *less* than said sum, we have supposed too much, and must find the difference between

the mean rate and the *lower* price, and then proceed as before; and finally prove our calculation by the Medial rule.

1st. A merchant wishes to mix wines worth 16sh. 18sh. and 22sh. so that the mixture may be worth 20 shillings; how much must he take of each sort?

	simples.		prices.	true proportion.		
2d diff.	1	at	16	1 at 16sh.	=	16
2	1	at	18	1 " 18	=	18
mean rate	1	at	22	3 " 22	=	66
required			56	5		100=20sh.
20 ×	3	.....	60			
1st difference too little			4=2			

Placing the simples at 1, and their respective prices at their side, and adding simples=3, and prices=56, we place the *mean rate* 20 on the left hand against the simples=3, which multiplied by 20=60; the sum of the prices being only 56, the 1st difference is 4 too little, therefore we must balance it by the difference of the mean rate and higher price; the mean rate being 20 and the higher price 22, the 2d difference is 2; now dividing the 1st difference by this 2d difference the quotient is 2, and we have acertained that to the simple standing against the price 22, we have to add 2, whereby we have the true proportion, above proved by Medial Alligation.

This simple rule is applicable to all cases. Having thus ascertained the proportion by the smallest quantity, we may change the mixture in the same proportion to any given quantity. Suppose it should be required to mix in the same proportion 500 gallons, we divide only by the sum of the smallest quantity 5, and say 500÷5=100, and we have

100	at	16	1600
100	"	18	1800
300	"	22	6600
500			10000=20 shillings.

From this illustration it is evident that all questions in

Alligation admit of a variety of mixtures and answers, but all these different answers must prove true by Medial Alligation, which will be fully explained by the following examples.

2d. A vintner would mix three kinds of wines, viz., 1 at 160 cts., 1 at 180 cts., and 1 at 240 cts. per gallon; how much of each sort must he take to make a mixture worth 190 cents?

	simples.	true proportion.	
2d diff. 10	1 at 160	1 at 160=160	
	1 at 180	2 at 180=360	
	1 at 240	1 at 240=240	
mean rate	580	4	760=190 cents.
190 ×	3 = 570		
1st difference	10 too large.		
	10=1.		

If we take here the mean rate 190, and that of 180 in simples, the 2d difference is also 10, and dividing by 10 into 10 the quotient is 1, wherefore we have to add 1 to the simple at 180, and have the true mixture, as proved above.

Suppose the same question in a different form, as,

	simples.	true proportion.	
2d diff. 30	1 at 160	4 at 160	640
	1 " 180	3 " 180	540
	1 " 240	3 " 240	720
mean rate	580	10	1900=190.
190× 3	570		
1st difference	$\frac{10}{30}=\frac{1}{3}$		

In taking now the differenceof the mean rate, 190, with 160, it is 30; the 2d difference, and now dividing by 30 into $10=\frac{1}{3}$ adding $\frac{1}{3}$ to the simple of 160 we have $1\frac{1}{3}$, 1 and 1; reducing this proportion to whole numbers in multiplying

each by 3 we have 4, 3, 3, simples for the true proportion, which must prove as above.

Let us again take the same question mixed, and proceed as

		simples.		true proportion.
1 diff.	30	1 at	160	simples.
"	10	1 at	180	5 at 160=800
2d "	40	1 at	240	5 at 180=900
mean rate			580	4 at 240=960
190 ×	3		570	14 2660=190.
1st difference			10	

In taking now the difference of 190 to 160=30
also 180=10
total 40

of both, dividing by 40 into 10 we have ¼ to each simple, and we have the proportion 1¼, 1¼ and 1 reduced to whole numbers=to 5, 5 and 4, which will prove by the true proportion.

It is evident by these illustrations that when we have once ascertained the true proportion of our ingredients, we may swell our calculations indefinitely. The question here analyzed is found in Stephen Pike's Arithmetic, page 165, question 2d, by which he gives 50 gallons at 160, 50 per gallon 180, and 40 gallons at 240; which is a variation in adding a cipher to the last proportion here given.

The rule of Alternate Alligation, as here laid down and explained, has great advantages and facilities over the rule generally taught in the present text books, by linking different quantities together; this process is not only tedious and complicated but also defective in itself, and fails to show the extensive application of this rule which renders it so valuable to the Merchant, Artist, and Chemist. Being persuaded of its importance, we will try to make it instructive and interesting both to teachers and learners by a variety of different examples.

3d. A storekeeper has several casks containing wine as follows: In the 1st casks he finds 26 gallons at $1 50 cts.; in the 2d, 12 gallons at 50 cts.; in the 3d, 19 gallons at $2; in the 4th, 22 gallons at $1 80 cts. which he wants to mix with wine of 80 cts. a gallon, so as to make a mixture worth $1 a gallon. How many gallons must he take of 80 cts. a gallon?

	Simples.		True Proportion.	
	26 at 150=	3900	26 at 150=	3900
	12 " 50=	600	12 " 50=	600
	19 " 200=	3800	19 " 200=	3800
	22 " 180=	3960	22 " 180=	3960
mean rate 100	1 " 80=	80	218 " 80=	17440
20, 2d dif.]	80	12340	297	29700=100
		8000		
	1st difference.	4340=217 gallons.		

Calculating the different simples of the remains of the wine in the casks with their prices, and 1 simple of the mixture, we find that 80 simples amount to $123 40 cts. but that the mean rate $100\times80=8000$ cts. we have for the 1st difference 4340; and as the 2d difference is 20 divided into 4340=217, and 1=218 gallons 80 cts. must be mixed with the remains, as shown by the proof above.

It will be observed that our rule is universal, but by other systems another process would be required to perform a simple question like this which does not present any points for linking;* but which will occur frequently, as mixtures are generally made by merchants and storekeepers, when commodities are nearly exhausted, and no longer worth their room in seperate lots.

4th. A merchant has three sorts of spirits of superior qualities, which he cannot sell at so low a price as the kind of spirit more frequently asked for, therefore he wishes to reduce the price to 80 cts. a gallon: his 1st quality is at

* See Lewis's Arithmetical Expositor, also Emerson's North American Arithmetic—*Alligation.*

\$2 50 cts. a gallon, 2d quality \$2 a gallon, and the 3d quality \$1 50 cts. a gallon. How much water must he mix to reduce the price of one gallon to 80 cts.?

	simples.		true proportion.	
	1 at 250		2 at 250=500	
	1 " 200		2 " 200=400	
2d dif. 80	1 " 150		2 " 150=300	
	0 " 0		9 " water=000	
mean rate	600	15	1200=80 cts.	
80 ×	3 =240			
1st difference	360=4½			

1st difference 360=4½ reduced to whole numbers gives the true proportion as proved above.

5th. How much sugar at 4 cents, 6 cents and 11 cents, must be mixed together to make a mixture worth 7 cents?

simples.	1 at 4
	1 " 6
	1 " 11
mean rate	21
7 ×	3 = 21

No difference, therefore an equal quantity of each sort.

6th. A goldsmith has gold of 17, 19, 21 and 24 carats fine, which he wants to melt together so as to make a compound of 22 carats fine; what proportion of each must he take?

	simples.		true proportion.
	1 at 17		2 at 17= 34
	1 " 19		2 " 19= 38
	1 " 21		2 " 21= 42
2d diff. 2	1 " 24		9 " 24=216
mean rate	81	15	330=22 carats fine
22×	4 = 88		

1st diff. too little 7=3½ reduced to whole numbers.

7th. A Goldsmith has gold at 15, 17, 20 and 22 carats fine, of which he wishes to make a mixture of 40 ounces of 18 carats fine; how much must he take of each sort?

	simples.		mixture required
	3	1 at 15	40=8
	1	1 " 17	true proportion.
2d diff.	4	1 " 20	simples.
		1 " 22	
mean rate		74	
	18 × 4	=72	
1st difference		2=½	

simples.		
12 at 15		180
12 " 17		204
8 " 20		160
8 " 22		176
40		720=18

Here an additional condition appears—the *quantity* of the compound is *fixed;* finding our 1st difference as before, we must change our simples so as to secure a factor of 40; if we take the 2d difference by 15 and 18, it is ⅔ added to the total simples 4=4⅔ which is no factor in 40, and in taking 17 and 18 it is 2, and add 2 to 4=6, which is also no factor of 40, but both differences added being 4 into the 1st difference 2=½ and ½ added to both prices=1 and 4=5; now dividing 5 in 40=8, by which the simples of the lowest proportion are multiplied, and will bring the required mixture of 40. As now 1½ by 8=12 as shown by the proportion above.

1½ "	=12
1 "	=8
1 "	=8
5	40

8th. How much gold at 14 and 16 carats fine, must be mixed with 6 ounces of 19, and 12 ounces of 22 carats fine, that the composition may be 20 carats fine?

		simples.
		6 at 19=114
		12 " 22=264
2d diff. 6	10	1 " 14= 14
2 is 4		1 " 16= 16
mean rate		408
20 ×	20	=400
	1st difference	8

Let us here inquire what proportional mixture would be the easiest to operate with, as 6 in $8=1\frac{1}{3}$, then 4 in $8=2$, or by both 10 in $8=\frac{4}{5}$; again, in dividing the 1st difference in two parts, as $6+2$ than 6 in $6=1$ and 4 in $2=\frac{1}{2}$, after which we would have the following perpetual simples.

1st.	2d.	3d.	4th.
simples.	simples.	simples.	simples.
6	6	6	6
12	12	12	12
$2\frac{1}{3}$	1	$1\frac{4}{5}$	2
1	3	$1\frac{4}{5}$	$1\frac{1}{2}$

All proportions thus established will answer the conditions of the question; here the 2d is the easiest, and the 3d is the most complicated.

We will therefore perform the operation by the second, and also by the third.

	true proportion. simples.			true proportion. simples.	
	6 at	$19=114$		6 at	$19=114$
	12 "	$22=264$		12 "	$22—264$
	1 "	$14=\ 14$		$1\frac{4}{5}$ "	$14=\ 25\frac{1}{5}$
mean rate	3 "	$16=\ 48$	mean rate	$1\frac{4}{5}$ "	$16=\ 28\frac{4}{5}$
20 ×	22	$440=20]$	20 ×	$21\frac{3}{5}$	$432=20$

In questions where the gross quantity is fixed, fractions cannot be changed to whole numbers, as illustrated by the 2d question; and for this reason we ought always to try to obtain whole numbers for the simples, if possible.

9th. What quantity of water at 1000 per ounce must be mixed with spirits of 850 degrees specific gravity, to make proof spirits of 925 degrees?

	simples.	
	1 at	1000
mean rate	1 "	850
925 ×	2	1850

a balance; no difference, therefore an equal quantity.

Spirits is lighter than water, and the mixture by weight would be the $\frac{850}{1000}$ of the specific gravity, reduced to its lowest term $\frac{17}{20}$; the mixture would be 20 lbs. or ounces of spirits to 17 lbs. or ounces of water, which would be an equal cube, or an equal quantity of measure.

We wish to make our *Calculator* equal to the times— The following question is addressed to the Washingtonians; and we claim for ourselves the merit of being the first who have applied formal *Alligation* to Cold Water.

Suppose in a hot day in July, the temperature of the Schuylkill water being at 90 degrees,
that of a Cistern . . . . . . 48 "
Ice Water . . . . 34 "
Ice . . . . 32 "

what proportion of each may we take to make a pleasant drink of 45 degrees?

11	1 at 90	Proportion.	
2d diff. 13	1 " 48	1 at 90=	90
24	1 " 34	1 " 48=	48
	1 " 32	2 " 34=	68
mean prop.	204	2 " 32=	64
45 ×	4 180	45×6	270=45
	1st diff. 24=1.		

These and similar questions are the most practical and useful in this rule, but we are also enabled to solve by it, a great many curious but useless questions of various true proportional answers.

EQUATION OF PAYMENTS.

This rule is similar with Medial Alligation. By Alligation we calculate the medium price of mixed commodities; and in the same manner we calculate the medium time, or interest, as the case may be; to do this we open an Interest Account like the following.

Interest Account of J. Smith.

Closed the 1st of July, at 6 per cent. per annum.

Debtor.				Creditor.			
Sums.	Commencement of Interest.	Number of days.	Proportional Numbers.	Sums.	Commencement of Interest.	Number of days.	Proportional Numbers.

According to the formula, all sums due or credited are entered and the time fixed when interest commences : from that date to the last of December the number of days are set down in that column, and multiplied by the sum for the proportional product. An Interest Account thus prepared may be finally calculated and closed in a few minutes, at any time or rate of interest whatever ; as also the equated time fixed if necessary, which the following example will fully illustrate.

Interest Account of Mr. Adam Good, of Philadelphia.
Closed the 20th of October, 1841, at 6 per cent.

Sums.		Commencement of Interest.			Number of days	Proportional Product.	Sums.	Commencement of Interest.			Number of days.	Proportional Product.
8000	"	1841	April	[illegible]	270	2160000	2000	1841	February	1	334	668000
6000	"	"	June	5	210	1260000	15100	1841	October	20	73	1102300
800	"	"	February	10	324	559200						
1700	"	"	October	1	92	156400						
600	"	"	October	26	67	40200			Balance			2105500
17100						3875800	17100					3875800

Per balance of interest on 2105500 proportional product by 6=$35 09 cts.

Illustration of the foregoing Account.

Supposing Mr. Good has received from a Merchant of New Orleans the following merchandise, viz.

The 2d January	1841,	\$8000 at	3 months'	credit.
" 1st February	"	6000 "	4 "	"
" 10th do.	"	800 "	cash.	
" 31st May	"	1700 "	4 months'	credit.
" 10th June	"	600 "	3 "	"

In adding the allowed credit with 3 days' grace to the date of the sale, the date where interest commences (which you will see in the interest account) we then set down the number of days to the last of December, and multiply the sum by it for the proportional product. An account thus prepared, which can be done at leisure, may at any time be closed at once, balanced and settled either by interest, or by the equated time if it admits.

In balancing the account we have 2105500 proportional product, which in dividing by 6000 or neglecting the cipher, by 6=to $3509\frac{1}{6}$=\$35 09 cts. interest. To know when the whole sum was due, or the equated time, we divide by the sum of 15100 into the proportional product of 2105500= $139\frac{66}{151}$ days, neglecting the fraction below $\frac{1}{2}$=139 days back from the 31st of December, will bring us to the 15th of August, the equated time; but as this date was before the 20th of October, we must charge Mr. Good interest on \$15100 for 139 days, thus

Statement.		Solution.	
Interest	\$15100		$151\cancel{0}\cancel{0}$
100	139 days.	$\cancel{1}\cancel{0}\cancel{0}$	139
360	6 interest.	6 $\cancel{3}\cancel{6}0$	$\cancel{0}$
		60	$20989=34, 98\frac{1}{6}$

the difference by neglecting the fraction of days $\frac{66}{151}$.

The equated time we see is the 15th of August, but as no settlement was made until the 20th of October, a

balance by equation of time can only be calculated and made by means of *interest*, and this on $15100 for 139 *days*, being the time from the 15th of August till the end of the year, and not till the 20th of October as it would appear; for that is the period to which Mr. Good has been credited with interest, as the credit of the interest account clearly shows.

The following calculation of an interest account will plainly show where equations of time can take place.

Interest Account of Mr. N. N. of P.
Closed on the 10th of June, 1841, at 6 per cent.

Sums.	Commencement of Interest.			Number of days.	Proportional Product.	Sums.	Commencement of Interest.			Number of days.	Proportional Product.
4000	1841	April	5	270	1080000	2000	1841	January	3	263	726000
4000	"	January	18	347	1388000	2000	"	Feb.	2	333	666000
6000	"	August	3	151	906000	13000	"	June	10	205	2665000
700	"	April	14	262	183400						
1700	"	Nov.	30	32	54400						
600	"	Sept.	11	112	67200						
per balance	"	June	10		378000						
17000					4057000	17000					4057000

Per balance of proportional product of 6 per cent. of $378000 \div 6 = \$6$ 30 cts.

But dividing the proportional product of 378000, by $13000 = 29$ days after the 10th of June is the equated time, viz. 8th July.

More on the subject of equations could be given, but the whole subject will be found analyzed and illustrated in my *Equation of Payments*, published by E. B. Clayton, No. 6 Tontine Buildings, Wall Street, New York.

STORAGE EQUATION.

Received, on account of N. N., at sundry times, various quantities of flour, as follows:

N. N. on storage, closed February 27th.

Dr.

Dates.		Quantities.	Days.	Proportional Product.	Dates.		Quantities.	Days.	Proportional Product.
January	13	200	353	70600	January	1	400	365	146000
February	1	218	334	72812	"	21	150	345	51750
"	27	250	308	77000	February	10	126	325	40950
"	27	210	308	64680	"	15	84	320	26880
per balance				17658	"	20	118	315	37170
		878		302750			878		302750
					From old account February	28	210	307	64470

By balance of 17658 proportional product at 30 days=589 bls, storage for 1 month.

Bought a lot of wheat of the following different weights for $1 20 a bushel, of 60lbs. per bushel; what is the amount?

Bushel.					lb.	lb.	oz.
100	at	64lbs.	4 oz.	982×100=	. . . . .	98200	
100	"	63 "	11 "	oz. 2×100=200÷16		=12	8
100	"	60 "	15 "			98212	8
100	"	61 "	10 "				
100	"	60 "	14 "				
100	"	59 "	10 "				
100	"	62 "	9 "				
100	"	61 "	5 "				
100	"	60 "	12 "				
100	"	59 "	7 "				
100	"	62 "	4 "				
100	"	61 "	8 "				
1260		738	13 Carried over.				

M

1260 " 738 " 13 "	Brought forward.				
100 " 59 " 2 "					
100 " 60 " 8 "					
100 " 61 " 4 "					
100 " 62 " 7 "				lbs.	oz.
982 " 2 "	. . . .	=		98212	8
$71\frac{1}{2}$ " 58 " 8 "	. . . .	=		4182	12
$1671\frac{1}{2}$ bushels.			total	102395	$\frac{1}{4}$

Statement.		Solution.	
how many $	$102395\frac{1}{4}$ lbs.		$102395\frac{1}{4}$
60	1 bushel.	$\cancel{60}$	1
1	120 cts.	1	$\cancel{120}$ 2
			$2047\ 90\frac{1}{2}$ Answer.

Therefore the amount of the whole lot $2047 $90\frac{1}{2}$ cts.

PROMISCUOUS QUESTIONS.

1st. Mr. Righteous's Will.

" All my property not related in my will shall be divided between my children in equal parts; but my bonds and mortgages below specified, amounting to $85,000, are to be divided in the following manner; each child arrived at the age of 21 years shall have an equal part, but those which may be at my death above 21 years, shall receive $500 for each year above that age, for the service rendered to the family. My neice, Mary, having faithfully assisted in the management of my family concerns, shall be entitled to the share of a child arrived at the age of 21 years. The said sum of $85,000 is to be divided at the rate of 5 per cent. simple interest."

The question is, how the sum of $85,000 must be divided among his five children and niece, which we may call A, B, C, D, E and M, if at the death of the father A was 25, B 23, C 21, D 16 and E 11 years of age?

A 4 years at \$500=\$2000
B 2 " " 500= 1000

$$\left.\begin{array}{l} \text{A 4 years at \$500=\$2000} \\ \text{B 2 " " 500= 1000} \end{array}\right\} \ldots \begin{array}{r} 85000 \\ 3000 \\ \hline 82000 \end{array}$$

$$\frac{82000}{82}=1000$$

A	15×	1000	=15000+2000	=17000
B	15 "	"	=15000+1000	=16000
C	15 "	"	=15000	=15000
D	12 "	"	=12000	=12000
E	10 "	"	=10000	=10000
M	15 "	"	=15000	=15000
	82			85000

Illustration. A had to receive 2000, and B 1000= 3000 must be taken from 85000=82000, which is to be proportionately divided, and of which A, B, C and M must receive an equal share, but D and E have only to receive so much when they are 21 years of age; to find their proportional stock we must use the rule of discount, and as the rate per cent is 5, and D is 5 years short of 21, and 5 years' interest and time 5×5=25 added to 100=125=100. What is the amount of one share?

$$\begin{array}{r|l} \text{share} & 1 \\ \cancel{125} & \cancel{100}\ \ 4 \\ 5 & \tfrac{4}{5}\ \text{Answer.} \end{array}$$

E lacks 10 years of 21, therefore 10×5=50 added to 100=150=100. What one share, thus:

$$\begin{array}{r|l} \text{share} & 1 \\ \cancel{150} & \cancel{100}\ \ 2 \\ 3 & \tfrac{2}{3}\ \text{Answer.} \end{array}$$

We have therefore A 1, B 1, C 1, D $\frac{4}{5}$, E $\frac{2}{3}$, M 1, reduced to whole numbers 15, 15, 15, 12, 10, 15, their proportional stock, as seen by the solution.

Now it must prove that \$12,000 put to interest for 5 years and \$10,000 for 10, at 5 per cent. must amount to \$15,000.

2d. A Stationer sold quills at \$1 40 cts. a thousand, $\frac{1}{4}$ of which was profit. When they became scarce he raised

the price to $1 47 cts. a thousand. How much per cent. did [he gain?

	Statement.		Solution.	
140				
$\frac{1}{4}$ = 35	per cent.	100		$\cancel{100}$ 20
buy'g pr. 105	b'g pr. 105	147 sel. pr.	$\cancel{105}$	
			$\cancel{21}$	$\cancel{147}$ 7
				140=40 per ct. Answer.

3d. A Merchant had 18000 lbs. of cotton, which he could sell at 9d. Virginia currency, but not finding a purchaser he bartered with A, and gave $7\frac{1}{2}$ lbs. of cotton for 2 yards of linen; not yet finding a cash purchaser he bartered his linen with B for coffee, and gave $2\frac{1}{2}$ yards of linen for $3\frac{1}{2}$ lbs. of coffee; and he changed his coffee with an Englishman for broadcloth, and gave 1 cwt. of coffee for 20 ells of broadcloth. Now he sold his broadcloth at $2 per yard. What is the loss or gain by this transaction, estimating the cotton at 9d. per lb. Virginia currency?

Statement.

$	18000lbs. cotton.
$7\frac{1}{2}$	2 yards linen.
$2\frac{1}{2}$	$3\frac{1}{2}$lbs, of coffee.
112	20 ells broadcloth.
4	5 yards.
1	$2

Solution.

	3
	$\cancel{18}000$
$\cancel{3}$ $\cancel{15}$	$\cancel{2}$
$\cancel{2}$	$\cancel{7}$
	$\cancel{20}$ $\cancel{4}$
$\cancel{5}$	$\cancel{5}$
$\cancel{2}$ $\cancel{14}$ $\cancel{112}$	$\cancel{2}$
	$\cancel{2}$
$\cancel{4}$	$\cancel{2}$
	$3000

Statement.

$	18000lbs. cotton.
1	9d. Va.
12	1sh.
20	1 £.
3	$10

Solution.

$\cancel{4}$ $\cancel{12}$	$1800\cancel{0}$=2250 Answer.
$\cancel{20}$	$\cancel{9}$ $\cancel{3}$
$\cancel{3}$	$\cancel{10}$ $\cancel{2}$
	difference $750 gain.

4th. A gentleman of the United States left his seven children a piece of land 6 miles long, and 4¾ miles broad. A son, settled in England, sold his part there for 4½£ sterling an acre, but being indebted to his relations he sent the money to the United States: exchange at that time 5 per cent. premium. For what amount ought the bill be drawn?

Statement.

$	$\frac{1}{7}$ of
	6 miles long.
	4¾ " broad.
miles sq. 1	640 acres.
acre 1	4½£. st.
£. st. 9	$40
100	105 premium.

Solution.

~~7~~	1
	~~6~~ 3
~~4~~	19
~~2~~	64~~0~~
~~9~~	~~9~~
	~~40~~
1~~00~~	~~105~~ 15
	$54720 Answer.

5th. The following history was related by a friend of mine when he proposed the question. A Frenchman had failed in trade by misfortune, but his father, being wealthy, resolved to establish him again in the United States, and furnished part of the cargo of a vessel and took passage with his son in her. Two sons of the father remained in France, also three children of his son; but two of them, brother and sister, had taken passage in a packet ship. The vessel with father and son was lost at sea. The cargo was insured in France for 1,347,840 francs, of which the grandfather possesed $\frac{1}{12}$ part. By chance a young clerk became acquainted with the brother and sister, his principal being appointed commissioner from the different partners of the insurance to settle the business in France, allowing him 4 per cent. commission. The clerk and brother having become intimate friends, he proposed to marry his sister and to commence business for themselves, by selling their parts of the insurance to said commissioner at a discount of 10 per cent.; and as they agreed so, the question is, with how

much federal money did the brothers-in-law begin business, if exchange at that time was 10½ d. sterling per franc, and from England to America 10 per cent. premium?

Statement.

$	$\frac{2}{5}$
	$\frac{1}{3}$
	$\frac{1}{12}$
	1347840 fs.
1	10½d. ster.
12	1sh.
20	1 £.
9	$40
100	110 premium.
110	100 discount.
104	100 commission.

Solution.

$\cancel{5}$	$\cancel{2}$		
$\cancel{3}$	1		
$\cancel{12}$	1		
$\cancel{2}$	$\cancel{1347840}$		$\cancel{449280}$
$\cancel{3}$ $\cancel{12}$		$\cancel{21}$ 7	$\cancel{37440}$
$\cancel{20}$		$\cancel{40}$	$\cancel{4160}$
$\cancel{5}$			40
$\cancel{100}$		$\cancel{100}$	$2800 Ans.
$\cancel{110}$		$\cancel{110}$	
$\cancel{104}$		$\cancel{100}$ $\cancel{5}$	

6th. A fox starts 80 yards before a greyhound, and is not perceived by him till he has been up 45 seconds; he scuds away at the rate of 9 miles an hour, and the hound pursues after him at the rate of 18 miles per hour. In how many seconds will the hound overtake the fox? and how far will each have run?

Recollecting the fox has 80 yards in advance, we must see, according to the conditions of the question, how many yards he has run in 45 seconds, thus:

Statement.

yds.	45 seconds.
60	1 minute.
60	1 hour.
1	9 miles.
1	1760 yds.

Solution.

	$\cancel{45}$ 9
$\cancel{4}$ $\cancel{20}$ $\cancel{60}$	$\cancel{5}$ $\cancel{3}$
$\cancel{2}$ $\cancel{60}$	$\cancel{1760}$ $\cancel{88}$ 22

Solution.

$\cancel{4}$ $\cancel{60}$	$\cancel{45}$ $\cancel{3}$	
	$\cancel{5}$	
$\cancel{2}$ $\cancel{60}$	$\cancel{1760}$ $\cancel{44}$ 22	80 adv.
	198 Answer . .	198

total advance before the hound starts 278 yds.

Now we will see in how many seconds the hound will gain 278 yards and overtake the fox.

Statement.

yds.	278 yds.
1760	1 mile.
9	1 hour.
1	60 minutes.
1	60 seconds.

Solution.

11 ~~22~~ ~~44~~ ~~1760~~	~~278~~ 139
~~3~~ ~~9~~	~~60~~ ~~2~~
	~~60~~ ~~20~~ 5
11	695($63\frac{2}{11}$ or

1 m. $3\frac{2}{11}$ seconds. Answer.

Finally, we have to ascertain how many yards the hound has run in $63\frac{2}{11}$ seconds.

Statement.

yds.	$63\frac{2}{11}$ seconds.
60	1 minute.
60	1 hour.
1	18 miles.
1	1760 yds.

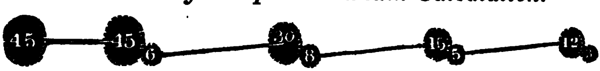
Solution.

~~11~~	~~695~~ 139
~~4~~ ~~60~~	~~18~~ ~~3~~
~~12~~ ~~60~~	~~1760~~ ~~44~~
	4
	556

556 yards the hound has run.
the fox had 80 " advance
476 the fox has run.

7th. Suppose a water wheel, of 45 feet diameter, turns 10 times round in 1 minute, having a similar wheel on the other end of its shaft, which runs in a wheel of 6 feet diameter, on whose shaft is a wheel of 30 feet diameter, which runs in a wheel of 8 feet diameter, which has a shaft with a wheel of 15 feet diameter, which runs in a wheel of 5 feet diameter, which also has a shaft holding a wheel of 12 feet diameter, which runs into one of 3 feet diameter. The question is, How often will the last wheel, of 3 feet diameter, turn round in 1 minute?

Illustration of Simple and Plain Calculation.



Regarding the figure representing the question to the eye, we see plainly that 3 is a divisor in 12, that 5 is a di-

visor in 15, and 8 a divisor in 30, and finally that 6 is a divisor in 45, which runs 10 times round in one minute. Our statement and solution are made easy, thus:

Statement.

3	12
5	15
8	30
6	45
	10 times.

Solution.

$\cancel{3}$	$\cancel{12}$ $\cancel{4}$
$\cancel{5}$	$\cancel{15}$
$\cancel{4}$ $\cancel{8}$	$\cancel{30}$ 15
$\cancel{6}$	45
	$\cancel{10}$ 5
	3375 times in 1 minute.

The proposition will be the same whether the measure is taken by the diameter or circumference; or by knowing the power of the water wheel, we may easily calculate the power of each following wheel, because the power decreases in the same ratio as the velocity increases.

Let us suppose that we know the power of the wheel to be 300 lbs., with what power would the smallest wheel revolve?

Here we only invert the terms, without regard to its velocity of 10 times in one minute, and we will obtain the proportionate power of the smallest wheel, thus:

Statement.

12	3
15	5
30	8
45	6
	300 lbs.

Solution.

$\cancel{2}$ $\cancel{12}$	$\cancel{3}$
$\cancel{15}$	$\cancel{5}$
$3\cancel{0}$	8
3 $\cancel{45}$	$\cancel{6}$
	$\cancel{300}$ $\cancel{2}$
9	$8 = \frac{8}{9}$ lbs. the

power of the wheel of 3 feet.

Should we want the power of the next smaller wheel, 5, we would have

Statement.

15	5
30	8
45	6
	300

Solution.

3 $\cancel{15}$	$\cancel{5}$
$3\cancel{0}$	8
$\cancel{3}$ $\cancel{45}$	$\cancel{6}$ 2
	$\cancel{300}$ 2
9	$32 = 3\frac{5}{9}$ lbs.

Here ocular demonstration will show to the smallest capacity, that if we leave out the wheels, 3 and 12, the power must increase 4 times, because 3 into $12=4$, and so is $4\times\frac{8}{9}=3\frac{5}{9}$. In like manner we may find either the velocity or power of each wheel by the instruction of the *Plain Calculator*.

Emerson's Prize Question, found in the third part of his Arithmetic, on the last page, has been taken from the original question proposed by Newton; but in changing $3\frac{1}{3}$ acres to $3\frac{1}{2}$ acres, the question, as proposed by Emerson, has become irrational, and the truth of the answer cannot be proved like that of Newton. I will therefore solve the original question as given by Newton, which runs thus:

If 12 oxen will eat $3\frac{1}{3}$ acres of grass in 4 weeks; and if 21 oxen will eat 10 acres of grass in 9 weeks; how many oxen will eat 24 acres in 18 weeks—the grass being allowed to grow uniformly?

The only difficulty of the question consists in finding out, by the two given suppositions, the proportional growth of the grass, it being rational with the number of oxen, as required by the question. It is plain by the conditions of the question, that the grass must grow uniformly, or proportionate to the number of acres and time. If we then suppose $3\frac{1}{3}$ acres have grown $\frac{1}{4}$ of itself in 4 weeks$=\frac{5}{6}$. We now say, if $3\frac{1}{3}$ acres have grown $\frac{5}{6}$ in 4 weeks, what will be the growth of 10 acres in 9 weeks?

Statement.

growth	10 acres.
$3\frac{1}{3}$	9 weeks.
4	$\frac{5}{6}$

Solution.

$\cancel{10}$	$\cancel{10}$
4	9
2 $\cancel{6}$	5
	$\cancel{3}$
8	$45=5\frac{5}{8}$

We have now to see if the rational growth of the grass will also be rational with the number of oxen given in the

second supposition, viz. 21 oxen; first, $\frac{5}{6}+3\frac{1}{3}=4\frac{1}{6}$, and second, $10+5\frac{5}{8}=15\frac{5}{8}$.

Now we have the following:

Statement.

oxen.	$15\frac{5}{8}$ acres.
$4\frac{1}{6}$	9 weeks, inverse.
inverse, 4	12 oxen.

Solution.

$$\begin{array}{r|l} \cancel{2}\ \cancel{8} & \cancel{125}\ 5 \\ \cancel{3}\ \cancel{9} & \cancel{12}\ 4 \\ \cancel{25} & \cancel{4} \\ & \cancel{6}\ \cancel{3} \\ \hline & 20 \text{ oxen.} \end{array}$$

According to the second supposition, the answer ought to be 21 oxen. We see, therefore, that the growth of the grass is irrational with the number of oxen required, and insufficient.

Increasing therefore by $\frac{1}{3}$ of $3\frac{1}{3}=1\frac{1}{9}$ growth in 4 weeks, what is the growth of 10 acres in 9 weeks?

Statement.

growth.	10 acres.
$3\frac{1}{3}$	9 weeks.
4	$1\frac{1}{9}$ growth.

Solution.

$$\begin{array}{r|l} \cancel{10} & \cancel{10} \\ 2\ \cancel{4} & \cancel{9} \\ \cancel{9} & \cancel{10}\ 5 \\ & 3 \\ \hline 2 & 15=7\frac{1}{2} \end{array}$$

Now $1\frac{1}{9}+3\frac{1}{3}=4\frac{4}{9}$, and $7\frac{1}{2}+10=17\frac{1}{2}$, we have the question—If 12 oxen eat $4\frac{4}{9}$ acres of grass in 4 weeks, how many oxen will eat $17\frac{1}{2}$ acres in 9 weeks?

Statement.

oxen.	$17\frac{1}{2}$ acres.
$4\frac{4}{9}$	9 weeks, inverse.
inverse 4	12 oxen.

Solution.

$$\begin{array}{r|l} \cancel{2} & \cancel{35}\ 7 \\ \cancel{9} & \cancel{12}\ \cancel{6}\ 3 \\ \cancel{2}\ \cancel{8}\ \cancel{40} & \cancel{9} \\ & \cancel{4} \\ \hline & 21 \text{ oxen.} \end{array}$$

The answer, 21 oxen, corresponding with the number required by the second supposition, we perceive that the proportional growth of the grass is rational for the two sup-

positions given by the author of the question. We have now to see if it will also be rational by the question, and if so, the questions are solved, and must prove true as has already been shown by Compound Proportion—that each of the questions proposed has at least five proof questions. We have the following question, by either of the two first suppositions, for the growth of the grass.

1st. If $3\frac{1}{3}$ acres of grass grow in 4 weeks $1\frac{1}{3}$, what will be the growth of 24 acres in 18 weeks?

Statement.

growth	24 acres.
$3\frac{1}{3}$	9 weeks.
4	$1\frac{1}{3}$

Solution.

$\cancel{10}$	$\cancel{24}$	6
$\cancel{4}$	$\cancel{18}$	2
$\cancel{9}$	$\cancel{10}$	
	3	
	36 growth.	
	24	
	60 acres inc. growth.	

2d. If 10 acres in 9 weeks grow $7\frac{1}{2}$, what will be the growth of 24 acres in 18 weeks?

Statement.

growth	24
10	18
9	$7\frac{1}{2}$

Solution.

$\cancel{2}$ $\cancel{10}$	$\cancel{24}$	12
$\cancel{9}$	$\cancel{18}$	
$\cancel{2}$	$\cancel{15}$	3
		36
		24
		60 acres inc. growth.

Finally, we have the question, How many oxen will eat 60 acres of grass, including growth, in 18 weeks?

1st. If 12 oxen will eat in 4 weeks $4\frac{4}{9}$ acres of grass, including growth, in 18 weeks.

2d. If 21 oxen will eat in 9 weeks $17\frac{1}{2}$ acres, including growth.

By either of these suppositions we will find the true answer by Inverse Proportion.

1st.	Statement.
oxen	60 acres.
$4\frac{4}{9}$	18 weeks, inverse.
Inverse, 4	12 oxen.

Solution.

$$\begin{array}{r|l} \cancel{2}\ \cancel{18} & \cancel{60}\ 3 \\ \cancel{40} & 12 \\ & \cancel{9} \\ & \cancel{4} \\ \hline & 36 \text{ oxen.} \end{array}$$

2d.	Statement.
oxen	60 acres.
$17\frac{1}{2}$	18 weeks, inverse.
Inverse, 9	21 oxen.

Solution.

$$\begin{array}{r|l} \cancel{18} & \cancel{60}\ 12 \\ \cancel{7}\ \cancel{35} & \cancel{21}\ \ 3 \\ & \cancel{2} \\ & \cancel{9} \\ \hline & \quad 36 \text{ oxen.} \end{array}$$

Now the three suppositions, including the proportional growth of the grass, are as follows:

1st. If 12 oxen eat up $4\frac{4}{9}$ acres of grass in 4 weeks, including the proportional growth, $1\frac{1}{9}$ acres.

2d. If 21 oxen eat up $17\frac{1}{2}$ acres of grass in 9 weeks, including the proportional growth, $7\frac{1}{2}$ acres.

3d. If 36 oxen eat up 60 acres of grass in 18 weeks, including the proportional growth, 36 acres.

They must all prove rational, if we take either of the suppositions, and omit one term of another, it must be found to answer, and the three suppositions give 18 questions, of which 12 are inverse and 6 direct.

The change made by Emerson, from $3\frac{1}{3}$ acres to $3\frac{1}{2}$, has entirely destroyed the rationality of the ingenious question of the famous author, and any answer to be found to it can only be an approximate one, which will be found to differ again with the others. If we let his $3\frac{1}{2}$ acres grow $1\frac{31}{60}$ in 4 weeks, 10 acres in 9 weeks will grow $9\frac{3}{4}$, and give for the second supposition $20\frac{300}{301}$ oxen, but not 21 oxen, as required; and so it will give for the question $37\frac{253}{395}$ oxen; and a great many different and irrational fractional answers may be found—among others that chosen by the Committee,

(out of 112 answers,) for an irrationally proposed question, which according to the key gives 37$\frac{113}{173}$ oxen for a true solution, which is also but an approximate one.

A wall is to be built 80 feet long, 45 feet high, and 2½ feet thick. How many bricks will it take, if each brick is 4½ inches in length, 4 inches broad, and 2½ thick?

Statement.

	bricks	80 feet long. 45 feet high. 2½ feet thick.
	cube foot.	1728 inches.
Inches.	long 4½ high 4 thick 2½	1 brick.

Solution.

		$\cancel{80}$ 40
		$\cancel{45}$ 5
	$\cancel{2}$	$\cancel{5}$
	$\cancel{9}$	1728
$\cancel{2}$	$\cancel{4}$	$\cancel{2}$
	$\cancel{5}$	$\cancel{2}$
		345600 brks., An.

A wall of 80 feet in length, 45 feet high, and 2½ feet thick, has been built with 345,600 bricks, of 4 inches broad and 2½ inches thick. What was the length of each brick in inches?

Statement.

		inches length	1 brick.	
		bricks 345600	0 foot long	inverse.
inverse.	broad	feet 45 inc. 12	4 in. broad 0 foot thick	inverse.
	thick	feet 2½ inc. 12	2½ in. do	inverse.
			80 feet 12 inches	long.

Solution.

$\cancel{4}$	$\cancel{2}$
$\cancel{5}$	$\cancel{80}$
$\cancel{345600}$	$\cancel{12}$ 9
$\cancel{360}$ $\cancel{2880}$	$\cancel{45}$
$\cancel{120}$ $\cancel{2}$	$\cancel{12}$ $\cancel{3}$
$\cancel{24}$	$\cancel{8}$
2	$\cancel{12}$
2	9 = 4½ in.

Answer.

It may be observed, if the result of a contraction for want of room cannot be placed on the side of the number contracted, it is sufficient if the result of it is either placed below or above, on the side of the perpendicular line to which it belongs.

The great end of accounts is to produce equality, and where this consummation is not possible by the real state of the case, we acknowledge the inequality as a balance. This balance constitutes our answers in all the various examples of our Arithmetics. In order to show that our assertion is neither fiction nor sophism, we add a plain example as formerly, and after the solution has been obtained, we present the statement anew, but place the answer or "balance" in the upper place of the left hand side, where its sign belongs. We then proceed to cancel the original terms of the question arbitrarily into the answer, or against each other, and if our calculation in the first instance has been correct, these antagonist quantities will completely annihilate each other—both sides will balance exactly.

Let us state the question. A merchant imported from England 975 ells of cloth, at 7s. 6d. per ell. The commission and duty amounted to 40 per cent., exchange 10 per cent. premium. For how many cents must one yard be sold in the United States to gain 12½ per cent.?

Statement.			Solution.				Proof.		
cts.	1 yard.		~~5~~	~~4~~		~~7~~ ~~77~~	~~231~~	~~4~~	
5	4 ells.		~~2~~	~~15~~	37		~~5~~	~~15~~	~~3~~
1	7½ sh.		~~9~~	~~2~~			~~2~~	~~2~~	
4½	1$		~~100~~	~~100~~			~~9~~	~~100~~	
1	100 cts.		~~100~~	~~140~~	7		~~100~~	~~140~~	~~2~~
100	140 duty.	~~25~~	~~100~~	11~~0~~			~~100~~	~~110~~	
100	110 prem.		~~2~~	~~225~~	~~9~~	~~4~~	~~100~~	~~225~~	~~25~~
100	112½ per cent.			231 cts.			~~2~~		

THE END.

PART II.

PROPERTIES OF FIGURES AND NUMBERS.

Besides the usual laws of calculation, laid down from early times and found in all arithmetical treatises, there are certain *natural affinities* existing between particular quantities or numbers, a proper use of which will render important assistance in obtaining correct results in accounts. When we consider the value of time, and our proneness to err in calculation, any thing which tends to shorten the operation and insure correctness is certainly worthy of attention; and this is the design of the following paragraphs.

It is plain that some quantities are more easily examined or calculated than others—thus small amounts are less difficult than large ones, even numbers are preferable to uneven, and those composed of units and ciphers are still more simple. These facts give value to those peculiarities above alluded to, as a knowledge of them enables us to change difficult numbers to easy ones. Thus, if we have to multiply 45 by 12, our *list of factors* shows us that 2 will divide any even number without a remainder; and the *list of easy multipliers* shows that any number ending with 5, if multiplied by an even number, will give a cipher at the end of the product, therefore we say

$$12 \div 2 = 6; \text{ also,}$$
$$45 \times 2 = 90$$

and 6 and 90 are more easily multiplied than 12 and 45, and less likely to be mistaken.

LIST OF FACTORS,* WITH THEIR SIGNS OR MARKS OF RECOGNITION.

The peculiarity of the figure 1 is, that standing singly it is powerless as a multiplier or divisor; it is of course a factor in all whole numbers.

2 is factor in all even figures and numbers, or will divide them without a remainder. Even numbers are those which end with 2, 4, 6, 8, 0.

3 is factor in all numbers of which the sum of the figures added horizontally can be divided by 3, as 11211; these figures added horizontally make 6, which can be divided by 3; therefore 11211 may be also.

4 is factor in all numbers of which the two last figures can be divided by 4 without a remainder, as 321112; here the two last figures, 12, may be divided by 4; therefore 321112 also may be.

5 is factor in all numbers of which the last figure is a 5 or a 0, as 3455 or 3360.

6 is factor in all *even numbers* which have the *token*† of 3.

7 has a variety of tokens, and also numerous *companion factors*,‡ all of great advantage in practice; they are as follow:

For 2 or 3 Figures.

1st. If the left hand figure or figures make the double of the right hand, 7 is factor; as 21, 42, 63, 84, 105, 126, 147, 168, 189.

2d. If the right hand figure is $\frac{1}{9}$th of the left hand figures, 7 is factor and 13 is companion factor, as 91, 182, 273.

For 3 or 4 Figures.

1st. If the left hand figure or figures make $\frac{1}{5}$th of the right hand, as 315, 735, 945, 525, 1155, 1575, 1365, 1785, &c., 7 is factor therein.

* *Factors* are smaller numbers contained an exact number of times in a larger.

† The peculiarity by which a number is known to be divisible by certain factors we call its *token* or sign.

‡ *Companion factors* are any *others* which will also divide a given number without a remainder—they are *companions* of the first named factor.

2d. If the two right hand figures make ⅓d of the left hand figure or figures, as 301, 903, 1204, 4515, 6622, &c. 7 is factor and 43 its companion factor.

3d. If any two similar figures enclose two ciphers, as in 1001, 9009, &c. 7 is its factor, and 11 and 13 are companion factors; as 7, 11, 13, multiplied to a continued product, give the sum 1001.*

For 5 *Figures.*

1st. If one cipher be enclosed by two equal figures, 7, 11, and 13, are factors.

2d. If the two right hand figures are the ⅓d of the three left hand ones, as 19866, 13545, 10836, &c., 7 is factor, and 43 companion factor.

For 5, 6, 7, or more figures, we may often readily discover the token by dividing the numbers into periods. Suppose 126,84, 315,63, 42,525, 3311,21, 630,1155; glancing at these numbers we may at once discover in the first two numbers the token in the three left hand and two right hand figures: in the third number we see the token in the two left hand and three right hand figures; in the fourth number we see as readily the token in the two right hand and the four left hand figures; as also in the last number the token is in the three first and four last figures, and we are therefore sure that 7 is factor. The various tokens of 7 are not so difficult to remember as at first may appear. Some attentive study and practice of *Contraction* and *Cancellation* will make the learner familiar with them.

8 is factor in all numbers of which the three last figures can be divided by 8, as 1267848, or 78594256.

Note.—If the number to be investigated have an even figure in the hundred place, see only if the two last figures will *cancel* by 8; or if the figure in the hundred place is an uneven figure, imagine it to be one hundred with the

* Observe, that 7, 11 and 13 are the factors of 1001. Suppose it should be required to multiply 789 by 7, 11 and 13, we would only have to repeat the figures of the multiplicand, and place them on one side of it, thus, 789789. *See Easy Multipliers and Divisors.*

two last figures, and you will discover it more easily, as 9776, imagining here the 7 to be a 1, you have 176, and see in an instant that these figures would divide by 8.

9 is factor in all numbers of which the sum of the figures added horizontally will divide by 9, as 121032, 34236, 98163; the first numbers added horizontally make 9, the second 18, and the third 27, &c., which sums divide by 9 without a remainder.

10 is factor in all numbers of which the last figure is a cipher.

11. Along with the figure 7, we have seen some of the numbers in which 11 is factor, but it has a general token for itself. It is factor in all numbers of which the figures subtracted from the left or right hand will leave no remainder, as 3861, if we subtract from the left hand to the right, and say 3 from 8=5, and 5 from 6=1, and 1 from 1=0; or from the right to the left hand, 1 from 6=5, and 5 from 8=3, and 3 from 3=0. If a figure cannot be subtracted from the next, add to it 11, and proceed as above; as 3267, say 3 from 2 we cannot, therefore 2+11=13, now 3 from 13 leaves 10, but 10 from 6 we cannot, therefore 6+11=17, and 10 from 17=7, and 7 from 7=0: also, from the right to the left hand, say 7 from 6 we cannot, therefore 6+11=17, and 7 from 17=10, now 10 from 2 we cannot, but 2+11=13, and 10 from 13=3, and 3 from 3=0.

The factor 11 may be often discovered if the figures of a number in the even places are equal to the uneven places; or if the difference is 11 or a compound of 11; as,

1st.	Places,	1	2	3	4	
	Numbers,	8	9	7	6	
2d.	Places,	1	2	3	4	5
	Numbers,	8	5	4	2	6

1st. Uneven places, 8+7=15 Even places, 9+6=15
2d. Uneven places, 8+4+6=18
Even places, 5+2=7, and 7 from 18=11 difference.

12 is a *composite factor** of $3\times 4=12$.

NOTE.—As a general remark, it is evident by analysis that the *products* of any factors discovered in a number are also *factors* of that number. We can easily analyse any number completely as soon as we find one factor of it. *Example:* Analyse 24024 to its prime factors.

$$\begin{array}{r|l} 8 & 24024 \\ 3 & 3003 \\ 7 & 1001 \\ 11 & 143 \\ & 13 \end{array}$$

8 is a *composite* factor, composed of $2\times 2\times 2$; therefore the prime factors are $2\times 2\times 2\times 3\times 7\times 11\times 13$.

The prime factors being 2, 2, 2, 3, 7, 11, 13; these multiplied together will give the number 24024, and therefore any combinations of them will also be factors in 24024.

13. We have seen in what number 13 is a companion factor with 7, but 13 is also a factor in numbers of three or 4 figures, if the one or two left hand figures divide in the two right hand ones, and give a quotient of 4, as 416, 728, 1248, 2392, &c.

14 is a composite factor of 2×7.

15 is a composite factor of 3×5.

16 is a composite factor of 2×8 or 4×4 or $2\times 2\times 2\times 2$.

17 is factor in numbers of two or three figures if the right hand figure divide in the left hand figure or figures, and give a quotient of 5, as 51, 255, 357, &c.

2d. Also in numbers of three or four figures, if the left hand figure or figures divide in the right hand figures and give a quotient of 2, as 612, 816, 918, 1326, 3774, 4386, &c.

3d. Also in numbers of four or five figures, if the first one or two figures divide in the three last and give a quotient of 3, as 1003, 6018, 12036, 45135, &c.

Observe, as a general rule, that if a factor have different tokens, we may discover the factor by dividing the number

* *Composite factors* are such as are formed by the multiplication of smaller ones which have been previously specified.

into periods. Suppose the numbers 816.3774 or 4386. 204.6018; by dividing them into periods as here by a point, we would find that 17 is factor in these numbers.

18 is a composite factor of 2×9.

19 is factor in all numbers of which the right hand figure if doubled, and the left hand figure added thereto, will give 19, or a compound of 19, as 19, 38, 57, 76, 95, 114, 133, 152, 171, &c. Let us examine if 19 is factor in 893: say $3+3=6$ and $89=95$, a compound of 19, viz. $5+5=10$ and $9=19$. Again, try 2793: say $3+3=6+9=15$, now $15+15=30+7=37$ taking 19 off$=18$, now $18+18=36+2=38$, a compound of 19, or repeating $8+8=16+3=19$.

20	is a composite factor of	$2\times2\times5$.
21	"	3×7.
22	"	2×11.
24	"	$2\times2\times2\times3$.
25	"	5×5; but it has also the

peculiarity of dividing any number the last two figures of which will divide by 25: thus in any number ending with 25, 50, 75, or two ciphers, 25 is factor.

26	is a composite factor of	2 and 13,	multiplied together.
27	"	3, 3, and 3,	"
28	"	2, 2, and 7,	"
30	"	2, 3, and 5,	"
32	"	2, 2, 2, 2, and 2,	"
33	"	3 and 11,	"
34	"	2 and 17,	"
35	"	5 and 7,	"
36	"	2, 2, 3, and 3,	"

37 is factor in numbers of three figures, if the figures are the same, viz. 111, 222, 333, &c. and the quotient will be the sum of these figures added horizontally, as first 3, second 6, third 9, &c. If more periods than one are found together in a number, the quotient is as readily found by placing a cipher between the sum of each. Thus in the number 999.666.555, you would discover the factor 37 at once, and your quotient would be 27018015, without performing the division, for the first period gives 27, then placing a cipher the se-

cond period is 18, place again a cipher and the third period is 15, and the quotient is obtained with certainty.

38	is a composite factor of	2 and 19,	multiplied together.
39	"	3 and 13,	"
40	"	2, 2, 2, and 5,	"
42	"	2, 3, and 7,	"

43 is factor in numbers of three, four, or five figures, if the left hand figure or figures are the treble of the right hand figures, as 301, 6622, 16555, without making the division the quotient will always prove to be the sum of the right hand period added to the double of the left hand; as first, $3+3=6+1=7$; second, $66+66=132+22=154$; third, $165+165=330+55=385$. In this property 7 is a companion factor.

44	is a composite factor of	2, 2 and 11,	multiplied together.
45	"	3, 3, and 5,	"
46	"	2 and 23,	"
48	"	2, 2, 2, 2, and 3,	"
49	"	7 and 7,	"
50	"	2, 5, and 5,	"
51	"	3 and 17,	"
52	"	2, 2, and 13,	"
54	"	2, 3, 3, and 3,	"
55	"	5 and 11,	"
56	"	2, 2, 2, and 7,	"
57	"	3 and 19,	"
58	"	2 and 29,	"

59 is factor in numbers of four or five figures, if the left hand figures divide in the three right hand ones, and give a quotient of 3; as 1003, 15045, 7021. Here we have 17 as companion factor.

60	"	2, 2, 3, and 5,	"
62	"	2 and 31,	"
63	"	3, 3, and 7,	"
64	"	2, 2, 2, 2, 2, and 2,	"
65	"	5 and 13,	"
66	"	2, 3, and 11,	"

67 is factor in numbers of three, four, or five figures, of which the left hand figure or figures make twice the

amount of the two right hand figures, as 201, 1005, 4422, 15075, 17889; we see here at a glance the token of 67, and we are also convinced that the addition of the two periods gives the true quotient without performing the division as in the first number $2+1=3$, $10+5=15$, $44+22=66$, $150+75=225$, $178+89=267$.

68	is a composite factor of	2, 2, & 17,	multiplied together.
69	"	3 and 23,	"
70	"	2, 5, and 7,	"
72	"	2, 2, 2, 3, and 3,	"
74	"	2 and 37,	"
75	"	3, 5, and 5,	"
76	"	2, 2, and 19,	"
77	"	7 and 11,	"
78	"	2, 3, and 13,	"
80	"	2, 2, 2, 2, and 5,	"
81	"	3, 3, 3, and 3,	"
82	"	2 and 41,	"
84	"	2, 2, 3, and 7,	"
85	"	5 and 17,	"
86	"	2 and 43,	"
87	"	3 and 29,	"
88	"	2, 2, 2, and 11,	"
90	"	2, 3, 3, and 5,	"
91	"	7 and 13,	"
92	"	2, 2, and 23,	"
93	"	3 and 31,	"
94	"	2 and 47,	"
95	"	5 and 19,	"
96	"	2, 2, 2, 2, 2, and 3,	"
98	"	2, 7, and 7,	"
99	"	3, 3, and 11,	"
100	"	2, 2, 5, and 5,	"

125 is factor in all numbers where 125, 250, 375, or 1000, are found at the end.

QUESTIONS AND ANSWERS ON THE FOREGOING.

Q. What is the token of 2? **A.** All even numbers.

Q. What are even numbers? **A.** All numbers which end with 2, 4, 6, 8, 0.

Q. What is the token of 3? **A.** If the sum of the figures of a number added horizontally can be divided by 3 without a remainder.

Q. What is the token of 4? **A.** If the two last figures can be divided by 4.

Q. What is the token of 5? **A.** If the last figure of a number is a 5 or a cipher.

Q. What is the token of 6? **A.** All *even numbers* which have the token of 3.

Q. What is the token of 7, for 2 or 3 figures? **A.** 1st. If the left hand figure or figures are the double of the right hand.

2d. If the right hand figure is $\frac{1}{9}$th of the left hand figure.

3d. If the left hand figure is $\frac{1}{5}$th of the right hand figure.

Q. What is the token of 7, for 4 figures in a number? **A.** 1st. If the *two* left hand figures are $\frac{1}{5}$th of the right hand ones.

2d. If the *two* right hand figures are $\frac{1}{3}$d of the two left hand ones.

3d. If two positive figures of the same name enclose two ciphers, as 1001.

Q. What is the token of 7, for 5 figures in a number? **A.** 1st. If one cipher is enclosed by two equal numbers.

2d. If the two right hand

figures are ⅓d of the three left hand ones.

Q. How are we to discover the token of 7 in larger numbers? A. By dividing those numbers into periods.

Q. What is the token of 8? A. If the three last figures of a number can be divided by 8.

Q. What is the token of 9? A. When the sum of the figures of a number added horizontally can be divided by 9.

Q. What is the token of 10? A. A cipher on the end of a number.

Q. What is the token of 11? A. 1st. If *two* ciphers are enclosed by two positive figures of the same name.
2d. If *one* cipher is enclosed by two positive figures of the same name.
3d. If the figures can be subtracted one from the other in regular progression horizontally and leave no remainder at the end.
4th. If the uneven places of a number added are equal to the even places, or show a difference of 11 or a compound of 11.

Q. What is the token of 13? A. 1st. If two ciphers are enclosed by two positive figures of the same name.
2d. If one cipher is enclosed as in answer 1.
3d. If the one or two left hand figures are ¼th of the two right hand ones.

Q. What is the token of 17? A. 1st. If the right hand figure is $\frac{1}{7}$th of the left hand one.

2d. If the left hand figure or figures are $\frac{1}{2}$ of the two right hand figures.

3d. If the left hand figure or figures are $\frac{1}{3}$d of the right hand figures.

Q. What is the token of 19? A. If the right hand figure doubled and added to the left hand one will make 19 or a compound of 19.

Q. What is the token of 25? A. All numbers which end with 25, 50, 75, or two ciphers.

Q. What is the token of 37? A. Any number composed of three figures of the same name, the addition of which horizontally is also the quotient of 37?

Q. What is the token of 43? A. If the two right had figures divide into the first, second, or third left hand figures, and give a quotient of three.

Q. What is the token of 59? A. If the one or two left hand figures divide in the right hand ones and give a quotient of three.

Q. What is the token of 67? A. If the two right hand figures are half of the one, two, or three left hand figures; when the addition of the periods gives the quotient.

Q. What is the token of 125? A. If 125, 250, 375, or three ciphers are on the end of a number.

EASY MULTIPLIERS AND DIVISORS

Are certain numbers which can be easily calculated, such as 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 31, 41, 51, 61, 71, 81, 91, 100, 101, 901, &c. 1000, 1001, 8001, &c.

There are also many difficult numbers which may be changed into easy ones by *analysis.* We have seen that the figures 1 and 0 are easy expressions, and if we can in a few moments change numbers composed of *other* figures into those of simpler characters, it is best to do so.

Suppose it is required to multiply or divide any number by 25 or 125; by analysis we know that $4\times 25=100$, and $8\times 125=1000$. Changing the multiplier or divisor to the power of 100 or 1000, and the multiplicand or dividend in the same proportion, the multiplication or division will be found easy, viz.

1. Multiply 12036 by 25; raising the multiplier by 4 to 100, we place two ciphers on the right hand side of the multiplicand, and divide by 4 for the answer. Thus, $1203600\div 4=300900$ Answer.*

2. Divide 78575 by 25; raising the divisor by 4 to 100, as also the dividend by 4, we have only to cut off two places for the result of the division. Thus,

$$\begin{array}{r} 78575 \\ 4 \\ \hline 3143(00 \end{array} \quad \text{Result, 3143.}$$

3. Multiply 6546784 by 125; raising the multiplier by 8 to 1000, and placing the three ciphers on the side of the multiplicand, and dividing by 8, we have the product; as $6546784000\div 8=818348000$, product. Or, dividing the

This *creation* of easy multipliers and divisors is based on the axiom that *equal* multiplication and division leave the original proportions the same.

same number by 125, raise the dividend by 8, and cut off 3 places, as

$$\begin{array}{r} 6546784 \\ 8 \\ \hline 52374(272 \end{array} \text{ Result.}$$

Or, $52374\frac{34}{125}$; the fraction of 1000 reduce by 8 to its lowest expression.

4. If we multiply any number by 10, 100, 1000, &c., we place the number of ciphers on the side of the multiplicand for the product; or, if we divide any number, we cut off as many figures as there are ciphers in the divisor for the quotient.

5. If we multiply by 11, we write the figures of the multiplicand one place to the right or left hand under it, and add for the product; as,

Multiply 3465 by 11, or

$$\begin{array}{r} 3465 \\ 3465 \\ \hline 38115 \end{array} \text{ product.} \qquad \begin{array}{r} 3465 \\ 3465 \\ \hline 38115 \end{array} \text{ product.}$$

6. If we multiply by 14, 17, &c. we multiply by 4 or 7 only, and place the product in the unit place, letting the multiplicand stand in the tens place, as the effect of the figure 1 of the multiplier.

Multiply 3465 by 14, also 3465 by 17

$$\begin{array}{r} 13860 \\ \hline 48510 \end{array} \text{ product.} \qquad \begin{array}{r} 24255 \\ \hline 58905 \end{array} \text{ product.}$$

7. If we have 102, 901, &c. for multipliers, we make the multiplicand occupy the *numerative place* of the figure 1 in the multiplier, acknowledge the effect of the cipher by placing it below according to its numerative place in the multiplier, and then proceed to multiply by the difficult figure, setting the first figure of its product in the numerative place of said multiplier, thus:

Multiply 3465 by 106 Multiply 3467 by 901

$$\begin{array}{r} 0 \\ 20790 \\ \hline 367290 \end{array} \text{ product.} \qquad \begin{array}{r} 312030 \\ \hline 3123767 \end{array} \text{ product.}$$

These principles have been long known and frequently applied in simple cases like the above, but we propose to extend the practice to large and intricate instances.

We have seen that any transposition of the factors of a number will not alter their joint product, nor will an exchange of factors disturb the result if it is done in accordance with true proportion. These principles enable us to deal with large amounts thus: Multiply

$$73678964 \text{ by } 1429 \times \underset{7|11}{\cancel{77}} \times \underset{4|7}{\cancel{28}} \times \underset{2|13}{\cancel{26}} \times 25 \times 5 \quad \text{new factors.}$$

Now we take the first multiplier 1429, and the first *new factor* 7, and their product is 10003; then we take of the remaining factors 7, 11, and 13, whose joint product is 1001. There are now left 4, 2, 25 and 5, which will be found to have a most brotherly affection for each other, as we may now see; $4 \times 2 \times 25 \times 5 = 1000$. So we now have, instead of that fearful array of figures with which we commenced, the following three good natured numbers called easy multipliers, viz. 10003, 1001, and 1000, which we use as follows:

$$\begin{array}{l} 73678964 \times 10003 \\ \quad\quad\quad 000 \\ \quad 221036892 \\ \hline 737010676892 \times 1001 \\ 73701067689200 \\ \hline 73774768756889\,2 \times 1000 = 737747687568892000 \end{array}$$

It is required to multiply 931 by $325 \times 44 \times 7$. Now $44 \div 4 = 11$, and $325 \times 4 = 1300$. We now find ourselves in possession of our old friends 7, 11, and 13, the last having brought with him two ciphers, to whose company we have no objection. Let us see how they will behave:

$$7 \times 11 \times 1300 = 100100$$

$$931 \times 100100 = 93193100.$$ (See note, p 3.)

The following are some of the numbers which, if multiplied together, will *produce* easy multipliers and divisors:

	34	37	67	167	267	367	467 by 3,
Produce	102	111	201	501	801	1101	1401

	25	26	27	28	75	175	275 by 4,
Produce	100	104	108	112	300	700	1100

All numbers ending with an even figure, if multiplied by 5, will have a cipher at the right hand of the product.

	17	18	19	35 by 6,
Produce	102	108	114	210

	16	43	143	243	443	1429 by 7,
Produce	112	301	1001	1701	3101	10003

	75	125	126	175	275 by 8,
Produce	600	1000	1008	1400	2200

	89	889	8889	88889 by 9,
Produce	801	8001	80001	800001;

or any number of 8's which have a 9 at the right hand.

Numbers, whose figures are such that a part of them are factors in the others, may be called easy multipliers. For instance, multiply 345 by 82; now we see at once that 2 is factor in 8, therefore we multiply by 2, and then multiply this product by 4, (placing the first figure of this last product in the tens place,*) and add these respective products for the final result: thus,

345×82
2 *units*
690×4 in the place of 8 *tens.*
2760
28290

Again: multiply 345 by 28.

345×28
2 *tens*
690×4 in the place of 8 *units.*
2760
9660

* The right hand figure of these secondary products must have the same numerative place as that of the original figure or number of the multiplier.

These very simple instances do not exhibit much advantage; they are meant to show the *operation* of the rule. We now proceed to more important cases.

Multiply 34675 by 189.

Usual way.	*Easy system.*
34675	34675 × 189
189	9 *units*
312075	312075 × 2 in the place of 18 *tens*.
277400	624150
34675	6553575 Result.
6553575 Result.	

Again: multiply 34675 by 918.

Usual way.	*Easy system.*
34675	34675 × 918
918	9 *hundreds*
277400	312075 × 2 in the place of 18 *units*.
34675	624150
312075	31831650
31831650	

There are a great many numbers which possess this inherent docility; the following are some of them:

123	189	312	412	497	639	749	896	1133	3311
124	213	315	416	535	642	756	918	1144	3612
126	214	318	424	545	648	763	927	1155	4411
142	216	321	426	546	654	784	936	1224	4812
147	217	324	427	549	672	816	945	1236	5511
153	218	327	428	567	714	819	954	1248	6012
155	243	328	432	568	721	824	963	1260	6611
162	246	332	436	612	726	834	968	1272	7212
164	248	336	448	618	728	847	972	1284	7711
168	273	357	455	624	729	856	981	1296	8811
182	279	364	459	636	735	864	1089	2211	9911
183	284	369	486	637	742	872	1122	2412	10100
186	287								

This table can be greatly extended, by merely placing ciphers between the measuring factors, thus:

Multiply 34567892 by 90036

9 *tens of thousands.*

3111110280000 × 4 in the place of 36 *units.*

1244444112

3112354724112

Illustration.—Having multiplied the number given by 9, which has the value of 90,000, we place four ciphers to the right hand of the product; and as 36÷9=4, we multiply the product of 9 by 4, and place it in the unit place. Let us now see the same figures in the multiplier, where the factor 9 is measured in the opposite direction.

Multiply 34567892 by 36009

9 *units*

311111028 × 4 in the place of 36 *thousands.*

1244444112000

1244755223028

We multiply by 9 units, and then multiply this product by 4 thousands, and add as before.

The following easy multipliers, embracing 1 and 0, are also *Prime Factors*, or original numbers: 101, 103, 107, 109, 401, 601, 701.

The importance of writing figures in their numerative places in all operations, cannot be too strictly enjoined. The following question of multiplication, performed by addition, will prove what is here advanced.

```
37807659 × 35872
-----------------
         75615318
    113422977
     189038295
      302461272
       264653613
-----------------
    1356236343648
```

Illustration.—1st. Adding the multiplicand to itself, we have the product of 2, placing the result exactly under the figure 2 of the multiplier.—2d. Adding the multiplicand

to the last sum or product, you have three times the multiplicand, which sum or product must be exactly placed under the multiplier 3.—3d. Now add the sum of 2 and 3, and place it exactly under the multiplier 5.—4th. Then add the sum of 5 and 3, and place that sum exactly under the multiplier 8; and finally add the sum of 2 and 5, and place the result under the figure 7 of the multiplier, and add the whole for the result.

This method of multiplication by addition will require much attention and skill, and is not recommended for universal practice; yet we can derive some advantages from it. Suppose we had the above multiplication performed by the usual method, and wished to prove the answer; we would add the product of 3 and 5, and see if the sum corresponded with the product of 8; if so, we should be pretty certain that in these three lines no error had taken place, because if a wrong calculation had occurred in both operations, the errors must have been such that the one exactly balanced the other, which will very seldom happen. If, also, the products of 2 and 5 are added, and correspond with the product of 7, it is scarcely possible that an error has been made; if the result is wrong, the fault must have been made in the final addition: the proof consists, therefore, in three additions. This proof of multiplication has the advantage over others of showing where the error is to be found.

ANALYTICAL TABLE OF FACTORS,

From 1 *to* 10,000,

ANALYSED TO THEIR PRIME FACTORS.

No.	Factors.	No.	Factors.	No.	Factors.
1	—	31	—	61	—
2	—	32	2. 2. 2. 2. 2	62	2. 31
3	—	33	3. 11	63	3. 3. 7
4	2. 2	34	2. 17	64	2. 2. 2. 2. 2. 2
5	—	35	5. 7	65	5. 13
6	2. 3	36	2. 2. 3. 3	66	2. 3. 11
7	—	37	—	67	—
8	2. 2. 2	38	2. 19	68	2. 2. 17
9	3. 3	39	3. 13	69	3. 23
10	2. 5	40	2. 2. 2. 5	70	2. 5. 7
11	—	41	—	71	—
12	2. 2. 3	42	2. 3. 7	72	2. 2. 2. 3. 3
13	—	43	—	73	—
14	2. 7	44	2. 2. 11	74	2. 37
15	3. 5	45	3. 3. 5	75	3. 5. 5
16	2. 2. 2. 2	46	2. 23	76	2. 2. 19
17	—	47	—	77	7. 11
18	2. 3. 3	48	2. 2. 2. 2. 3	78	2. 3. 13
19	—	49	7. 7.	79	—
20	2. 2. 5	50	2. 5. 5	80	2. 2. 2. 2. 5
21	3. 7	51	3. 17	81	3. 3. 3. 3
22	2. 11	52	2. 2. 13	82	2. 41
23	—	53	—	83	—
24	2. 2. 2. 3	54	2. 3. 3. 3.	84	2. 2. 3. 7
25	5. 5	55	5, 11	85	5. 17
26	2. 13	56	2. 2. 2. 7	86	2. 43
27	3. 3. 3	57	3. 19	87	3. 29
28	2. 2. 7	58	2. 29	88	2. 2. 2. 11
29	—	59	—	89	—
30	2. 3. 5	60	2. 2. 3. 5	90	2. 3. 3. 5

No.	Factors.	No.	Factors.	No.	Factors.
91	7. 13	131	—	171	3. 3. 19
92	2. 2. 23	132	2. 2. 3. 11	172	2. 2. 43
93	3. 31	133	7. 19	173	—
94	2. 47	134	2. 67	174	2. 3. 29
95	5. 19	135	3. 3. 3. 5	175	5. 5. 7
96	2. 2. 2. 2. 2. 3	136	2. 2. 2. 17	176	2. 2. 2. 2. 11
97	—	137	—	177	3. 59
98	2. 7. 7	138	2. 3. 23	178	2. 89
99	3. 3. 11	139	—	179	—
100	2. 2. 5. 5	140	2. 2. 5. 7	180	2. 2. 3. 3. 5
101	—	141	3. 47	181	—
102	2. 3. 17	142	2. 71	182	2. 7. 13
103	—	143	11. 13	183	3. 61
104	2. 2. 2. 13	144	2. 2. 2. 2. 3. 3	184	2. 2. 2. 23
105	3. 5. 7	145	5. 29	185	5. 37
106	2. 53	146	2. 73	186	2. 3. 31
107	—	147	3. 7. 7	187	11. 17
108	2. 2. 3. 3. 3	148	2. 2. 37	188	2. 2. 47
109	—	149	—	189	3. 3. 3. 7
110	2. 5. 11	150	2. 3. 5. 5	190	2. 5. 19
111	3. 37	151	—	191	—
112	2. 2. 2. 2. 7	152	2. 2. 2. 19	192	2.2.2.2.2.2.3
113	—	153	3. 3. 17	193	—
114	2. 3. 19	154	2. 7. 11	194	2. 97
115	5. 23	155	5. 31	195	3. 5. 13
116	2. 2. 29	156	2. 2. 3. 13	196	2. 2. 7. 7
117	3. 3. 13	157	—	197	—
118	2. 59	158	2. 79	198	2. 3. 3. 11
119	7. 17	159	3. 53	199	—
120	2. 2. 2. 3. 5	160	2. 2. 2. 2. 2. 5	200	2. 2. 2. 5. 5
121	11. 11	161	7. 23	201	3. 67
122	2. 61	162	2. 3. 3. 3. 3	202	2. 101
123	3. 41	163	—	203	7. 29
124	2. 2. 31	164	2. 2. 41	204	2. 2. 3. 17
125	5. 5. 5	165	3. 5. 11	205	5. 41
126	2. 3. 3. 7	166	2. 83	206	2. 103
127	—	167	—	207	3. 3. 23
128	2.2.2.2.2.2.2	168	2. 2. 2. 3. 7	208	2. 2. 2. 2. 13
129	3. 43	169	13. 13	209	11. 19
130	2. 5. 13	170	2. 5. 17	210	2. 3. 5. 7

No.	Factors.	No.	Factors.	No.	Factors.
211	—	251	—	291	3. 97
212	2. 2. 53	252	2. 2. 7. 9	292	2. 2. 73
213	3. 71	253	11. 23	293	—
214	2. 107	254	2. 127	294	2. 3. 7. 7
215	5. 43	255	3. 5. 17	295	5. 59
216	2. 2. 2. 3. 3. 3	256	2.2.2.2.2.2.2.2	296	2. 2. 2. 37
217	7. 31	257	—	297	3. 3. 3. 11
218	2. 109	258	2. 3. 43	298	2. 149
219	3. 73	259	7. 37	299	13. 23
220	2. 2. 5. 11	260	2. 2. 5. 13	300	2. 2. 3. 5. 5
221	13. 17	261	3. 3. 29	301	7. 43
222	2. 3. 37	262	2. 131	302	2. 151
223	—	263	—	303	3. 101
224	2. 2. 2. 2. 2. 7	264	2. 2. 2. 3. 11	304	2. 2. 2. 2. 19
225	3. 3. 5. 5	265	5. 53	305	5. 61
226	2. 113	266	2. 7. 19	306	2. 3. 3. 17
227	—	267	3. 89	307	—
228	2. 2. 3. 19	268	2. 2. 67	308	2. 2. 7. 11
229	—	269	—	309	3. 103
230	2. 5. 13	270	2. 3. 3. 3. 5	310	2. 5. 31
231	3. 7. 11	271	—	311	—
232	2. 2. 2. 29	272	2. 2. 2. 2. 17	312	2. 2. 2. 3. 13
233	—	273	3. 7. 13	313	—
234	2. 3. 3. 13	274	2. 137	314	2. 157
235	5. 47	275	5. 5. 11	315	3. 3. 5. 7
236	2. 2. 59	276	2. 2. 3. 23	316	2. 2. 79
237	3. 79	277	—	317	—
238	2. 7. 17	278	2. 139	318	2. 3. 53
239	—	279	3. 3. 31	319	11. 29
240	2. 2. 2. 2. 3. 5	280	2. 2. 2. 5. 7	320	2.2.2.2.2.2.5
241	—	281	—	321	3. 107
242	2. 11. 11	282	2. 3. 47	322	2. 7. 23
243	3. 3. 3. 3. 3	283	—	323	17. 19
244	2. 2. 61	284	2. 2. 71	324	2. 2. 3. 3. 3. 3
245	5. 7. 7	285	3. 5. 19	325	5. 5. 13
246	2. 3. 41	286	2. 11. 13	326	2. 163
247	13. 19	287	7. 41	327	3. 109
248	2. 2. 2. 31	288	2.2.2.2.2.3.3	328	2. 2. 2. 41
249	3. 83	289	17. 17	329	7. 47
250	2. 5. 5. 5	290	2. 5. 29	330	2. 3. 5. 1

No.	Factors.	No.	Factors.	No.	Factors.
331	—	371	7. 53	411	3. 137
332	2. 2. 83	372	2. 2. 3. 31	412	2. 2. 103
333	3. 3. 37	373	—	413	7. 59
334	2. 167	374	2. 11. 17	414	2. 3. 3. 23
335	5. 67	375	3. 5. 5. 5	415	5. 83
336	2. 2. 2. 2. 3. 7	376	2. 2. 2. 47	416	2. 2. 2. 2. 2. 13
337	—	377	13. 29	417	3. 139
338	2. 13. 13	378	2. 3. 3. 3. 7	418	2. 11. 19
339	3. 113	379	—	419	—
340	2. 2. 5. 17	380	2. 2. 5. 19	420	2. 2. 3. 5. 7
341	11. 31	381	3. 127	421	—
342	2. 3. 3. 19	382	2. 191	422	2. 211
343	7. 7. 7	383	—	423	3. 3. 47
344	2. 2. 2. 43	384	2.2.2.2.2.2.2.3	424	2. 2. 2. 53
345	3. 5. 23	385	5. 7. 11	425	5. 5. 17
346	2. 173	386	2. 193	426	2. 3. 71
347	—	387	3. 3. 43	427	7. 61
348	2. 2. 3. 29	388	2. 2. 97	428	2. 2. 107
349	—	389	—	429	3. 11. 13
350	2. 5. 5. 7	390	2. 3. 5. 13	430	2. 5. 43
351	3. 3. 3. 13	391	17. 23	431	—
352	2. 2. 2. 2. 2. 11	392	2. 2. 2. 7. 7	432	2. 2. 2. 2. 3. 3. 3
353	—	393	3. 131	433	—
354	2. 3. 59	394	2. 197	434	2. 7. 31
355	5. 71	395	5. 79	435	3. 5. 29
356	2. 2. 89	396	2. 2. 3. 3. 11	436	2. 2. 109
357	3. 7. 17	397	—	437	19. 23
358	2. 179	398	2. 199	438	2. 3. 73
359	—	399	3. 7. 19	439	—
360	2. 2. 2. 3. 3. 5	400	2. 2. 2. 2. 5. 5	440	2. 2. 2. 5. 11
361	19. 19	401	—	441	3. 3. 7. 7
362	2. 181	402	2. 3. 67	442	2. 13. 17
363	3. 11. 11	403	13. 31	443	—
364	2. 2. 7. 13	404	2. 2. 101	444	2. 2. 3. 37
365	5. 73	405	3. 3. 3. 3. 5	445	5. 89
366	3. 2. 61	406	2. 7. 29	446	2. 223
367	—	407	11. 37	447	3. 149
368	2. 2. 2. 2. 23	408	2. 2. 2. 3. 17	448	2. 2. 2. 2. 2. 2. 7
369	3. 3. 41	409	—	449	—
370	2. 5. 37	410	2. 5. 41	450	2. 3. 3. 5. 5

No.	Factors.	No.	Factors.	No.	Factors.
451	11. 41	491	—	531	3. 3. 59
452	2. 2. 113	492	2. 2. 3. 41	532	2. 2. 7. 19
453	3. 151	493	17. 29	533	13. 41
454	2. 227	494	2. 13. 19	534	2. 3. 89
455	5. 7. 13	495	3. 3. 5. 11	535	5. 107
456	2. 2. 2. 3. 19	496	2. 2. 2. 2. 31	536	2. 2. 2. 67
457	—	497	7. 71	537	3. 179
458	2. 229	498	2. 3. 83	538	2. 269
459	3. 3. 3. 17	499	—	539	7. 7. 11
460	2. 2. 5. 23	500	2. 2. 5. 5. 5	540	2. 2. 3. 3. 3. 5
461	—	501	3. 167	541	—
462	2. 3. 7. 11	502	2. 251	542	2. 271
463	—	503	—	543	3. 181
464	2. 2. 2. 2. 29	504	2. 2. 2. 7. 9	544	2.2.2.2.2.17
465	3. 5. 31	505	5. 101	545	5. 109
466	2. 223	506	2. 11. 23	546	2. 3. 7. 13
467	—	507	3. 13. 13	547	—
468	2. 2. 3. 3. 13	508	2. 2. 127	548	2. 2. 137
469	7. 67	509	—	549	3. 3. 61
470	2. 5. 47	510	2. 3. 5. 17	550	2. 5. 5. 11
471	3. 157	511	7. 73	551	19. 29
472	2. 2. 2. 59	512	2.2.2.2.2 2222	552	2. 2. 2. 3. 23
473	11. 43	513	3. 3. 3. 19	553	7. 79
474	2. 3. 79	514	2. 257	554	2. 277
475	5. 5. 19	515	5. 103	555	5. 3. 37
476	2. 2. 7. 17	516	2. 2. 3. 43	556	2. 2. 139
477	3. 3. 53	517	11. 47	557	—
478	2. 239	518	2. 7. 37	558	2. 3. 3. 31
479	—	519	3. 173	559	13. 43
480	2.2.2.2.2. 3. 5	520	2. 2. 2. 5. 13	560	2. 2. 2. 2. 5. 7
481	13. 37	521	—	561	3. 11. 17
482	2. 241	522	2. 3. 3. 29	562	2. 281
483	3. 7. 23	523	—	563	—
484	2. 2. 11. 11	524	2. 2. 131	564	2. 2. 3. 47
485	5. 97	525	3. 5. 5. 7	565	5. 113
486	2. 3. 3. 3. 3. 3.	526	2. 263	566	2. 283
487	—	527	17. 31	567	3. 3. 3. 3. 7
488	2. 2. 2. 61	528	2.2.2.2. 3. 11	568	2. 2. 2. 71
489	3. 163	529	23. 23.	569	—
490	2. 5. 7. 7	530	2. 5. 53	570	2. 3. 5. 19

No.	Factors.	No.	Factors.	No.	Factors.
571	—	611	13. 47	651	3. 7. 31
572	2. 2. 11. 13	612	2. 2. 3. 3. 17	652	2. 2. 163
573	3. 191	613	—	653	—
574	2. 7. 41	614	2. 307	654	2. 3. 109
575	5. 5. 23	615	3. 5. 41	655	5. 131
576	2.2.2.2.2.2.3.3	616	2. 2. 2. 7. 11	656	2. 2. 2. 2. 41
577	—	617	—	657	3. 3. 73
578	2. 17. 17	618	2. 3. 103	658	2. 7. 47
579	3. 193	619	—	659	—
580	2. 2. 5. 29	620	2. 2. 5. 31	660	2. 2. 3. 5. 11
581	7. 83	621	3. 3. 3. 23	661	—
582	2. 3. 97	622	2. 311	662	2. 331
583	11. 53	623	7. 89	663	3. 13. 17
584	2. 2. 2. 73	624	2.2.2.2.3.13	664	2. 2. 2. 83
585	3. 3. 5. 13	625	5. 5. 5. 5.	665	5. 7. 19
586	2. 293	626	2. 313	666	2. 3. 3. 37
587	—	627	3. 11. 19	667	23. 29
588	2. 2. 3. 7. 7	628	2. 2. 157	668	2. 2. 167
589	19. 31	629	17. 37	669	3. 223
590	2. 5. 59	630	2. 3. 3. 5. 7	670	2. 5. 67
591	3. 197	631	—	671	11. 61
592	2. 2. 2. 2. 37	632	2. 2. 2. 79	672	2.2.2.2.2.3.7
593	—	633	3. 211	673	—
594	2. 3. 3. 3. 11	634	2. 317	674	2. 337
595	5. 7. 17	635	5. 127	675	3. 3. 3. 5. 5
596	2. 2. 149	636	2. 2. 3. 53	676	2. 2. 13. 13
597	3. 199	637	7. 7. 13	677	—
598	2. 13. 23	638	2. 11. 29	678	2. 3. 113
599	—	639	3. 3. 71	679	7. 97
600	2. 2. 2. 3. 5. 5	640	2.2.2.2.2.2.2.5	680	2. 2. 2. 5. 17
601	—	641	—	681	3. 227
602	2. 7. 43	642	2. 3. 107	682	2. 11. 31
603	3. 3. 67	643	—	683	—
604	2. 2. 151	644	2. 2. 7. 23	684	2. 2. 3. 3. 19
605	5. 11. 11	645	3. 5. 43	685	5. 137
606	2. 3. 101	646	2. 17. 19	686	2. 7. 7. 7
607	—	647	—	687	3. 229
608	2.2.2.2.2.19	648	2.2.2.3.3.3.3	688	2. 2. 2. 2. 43
609	3. 7. 29	649	11. 59	689	13. 53
610	2. 5. 61	650	2. 5. 5. 13	690	2. 3. 5. 23

No.	Factors.	No.	Factors.	No.	Factors.
691	—	731	17. 43	771	3. 257
692	2. 2. 173	732	2. 2. 3. 61	772	2. 2. 193
693	3. 3. 7. 11	733	—	773	—
694	2. 347	734	2. 367	774	2. 3. 3. 43
695	5. 139	735	3. 5. 7. 7	775	5. 5. 31
696	2. 2. 2. 3. 29	736	2.2.2.2.2.23	776	2. 2. 2. 97
697	17. 41	737	11. 67	777	3. 7. 37
698	2. 349	738	2. 3. 3. 41	778	2. 389
699	3. 233	739	—	779	19. 41
700	2. 2. 5. 5. 7	740	2. 2. 5. 37	780	2. 2. 3. 5. 13
701	—	741	3. 13. 19	781	11. 71
702	2. 3. 3. 3. 13	742	2. 7. 53	782	2. 17. 23
703	19. 37	743	—	783	3. 3. 3. 29
704	2.2.2.2.2.2.11	744	2. 2. 2. 3. 31	784	2. 2. 2. 2. 7. 7
705	3. 5. 47	745	5. 149	785	5. 157
706	2. 353	746	2. 373	786	2. 3. 131
707	7. 101	747	3. 3. 83	787	—
708	2. 2. 3. 59	748	2. 2. 11. 17	788	2. 2. 197
709	—	749	7. 107	789	3. 263
710	2. 5. 71	750	2. 3. 5. 5. 5.	790	2. 5. 79
711	3. 3. 79	751	—	791	7. 113
712	2. 2. 2. 89	752	2. 2. 2. 2. 47	792	2.2.2.3.3.11
713	23. 31	753	3. 251	793	13. 61
714	2. 3. 7. 17	754	2. 13. 29	794	2. 397
715	5. 11. 13	755	5. 151	795	3. 5. 53
716	2. 2. 179	756	2.2.3.3.3.7.	796	2. 2. 199
717	3. 239	757	—	797	—
718	2. 359	758	2. 379	798	2. 3. 7. 19
719	—	759	3. 11. 23	799	17. 47
720	2.2.2.2. 3.3.5	760	2. 2. 2. 5. 19	800	2.2.2.2.2.5.5
721	7. 103	761	—	801	3. 3. 89
722	2. 19. 19	762	2. 3. 127	802	2. 401
723	3. 241	763	7. 109	803	11. 73
724	2. 2. 181	764	2. 2. 191	804	2. 2. 3. 67
725	5. 5. 29	765	3. 3. 5. 17	805	5. 7. 23
726	2. 3. 11. 11	766	2. 383	806	2. 13. 31
727	—	767	13. 59	807	3. 269
728	2. 2. 2. 7. 13	768	22222222.3	808	2. 2. 2. 101
729	3. 3. 3. 3. 3. 3	769	—	809	—
730	2. 5. 73	770	2. 5. 7. 11	810	2. 3. 3. 3. [illegible]

No.	Factors.	No.	Factors.	No.	Factors.
811	—	851	23. 37	891	3. 3. 3. 3. 11
812	2. 2. 7. 29	852	2. 2. 3. 71	892	2. 2. 223
813	3. 271	853	—	893	19. 47
814	2. 11. 37	854	2. 7. 61	894	2. 3. 149
815	5. 163	855	3. 3. 5. 19	895	5. 179
816	2.2.2.2.3.17	856	2. 2. 2. 107	886	2.2.2.2.2.2.2.7
817	19. 43	857	—	897	3. 13. 23
818	2. 409	858	2. 3. 11. 13	898	2. 449
819	3. 3. 7. 13	859	—	899	29. 31
820	2. 2. 5. 41	860	2. 2. 5. 43	900	2. 2. 3. 3. 5. 5
821	—	861	3. 7. 41	901	17. 53
822	2. 3. 137	862	2. 431	902	2. 11. 41
823	—	863	—	903	3. 7. 43
824	2. 2. 2. 103	864	2.2.2.2.2.3.3.3	904	2. 2. 2. 113
825	3. 5. 5. 11	865	5. 173	905	5. 181
826	2. 7. 59	866	2. 433	906	2. 3. 151
827	—	867	3. 17. 17	907	—
828	2. 2. 3. 3. 23	868	2. 2. 7. 31	908	2. 2. 227
829	—	869	11. 79	909	3. 3. 101
830	2. 5. 83	870	2. 3. 5. 29	910	2. 5. 7. 13
831	3. 277	871	—	911	—
832	2.2.2.2.2.2.13	872	2. 2. 2. 109	912	2.2.2.2.3.19
833	7. 7. 17	873	3. 3. 97	913	11. 83
834	2. 3. 139	874	2. 19. 23	914	2. 457
835	5. 167	875	5. 5. 5. 7	915	3. 5. 61
836	2. 2. 11. 19	876	2. 2. 3. 73	916	2. 2. 229
837	3. 3. 3. 31	877	—	917	7. 131
838	2. 419	878	2. 439	918	2. 3. 3. 3. 17
839	—	879	3. 293	919	—
840	2. 2. 2. 3. 5. 7	880	2.2.2.2.5.11	920	2. 2. 2. 5. 23
841	29. 29	881	—	921	3. 307
842	2. 421	882	2. 3. 3. 7. 7	922	2. 461
843	3. 281	883	—	923	13. 71
844	2. 2. 211	884	2. 2. 13. 17	924	2. 2. 3. 7. 11
845	5. 13. 13	885	3. 5. 59	925	5. 5. 37
846	2. 3. 3. 47	886	2. 443	926	2. 463
847	7. 11. 11	887	—	927	3. 3. 103
848	2. 2. 2. 2. 53	888	2. 2. 2. 3. 37	928	2.2.2.2.2.29
849	3. 283	889	7. 127	929	—
850	2. 5. 5. 17	890	2. 5. 89	930	2. 3. 5. 31

No.	Factors.	No.	Factors.	No.	Factors.
931	7. 7. 19	971	—	1037	17. 61
932	2. 2. 233	972	2.2.3.3.3.3.3	1039	—
933	3. 311	973	7. 139	1043	7. 149
934	2. 467	974	2. 487	1049	—
935	5. 11. 17	975	3. 5. 5. 13	1051	—
936	2. 2. 2.3.3. 13	976	2. 2. 2. 2. 61	1057	7. 151
937	—	977	—	1061	—
938	2. 7. 67	978	2. 3. 163	1063	—
939	3. 313	979	11. 89	1067	11. 97
940	2. 2. 5. 47	980	2. 2. 5. 7. 7	1069	—
941	—	981	3. 3. 109	1073	29. 37
942	2. 3. 157	982	2. 491	1079	13. 83
943	23. 41	983	—	1081	23. 47
944	2. 2. 2. 2. 59	984	2. 2. 2. 3. 41	1087	—
945	3. 3. 3. 5. 7	985	5. 197	1091	—
946	2. 11. 43	986	2. 17. 29	1093	—
947	—	987	3. 7. 47	1097	—
948	2. 2. 3. 79	988	2. 2. 13. 19	1099	7. 157
949	13. 73	989	23. 43	1103	—
950	2. 5. 5. 19	990	2. 3. 3. 5. 11	1109	—
951	3. 317	991	—	1111	11. 101
952	2. 2. 2. 7. 17	992	2.2.2.2.2.31	1117	—
953	—	993	3. 331	1121	19. 59
954	2. 3. 3. 53	994	2. 7. 71	1123	—
955	5. 191	995	5. 199	1127	7. 7. 23
956	2. 2. 239	996	2. 2. 3. 83	1129	—
957	3. 11. 29	997	—	1133	11. 103
958	2. 479	998	2. 499	1139	17. 67
959	7. 137	999	3. 3. 3. 37	1141	7. 163
960	2.2.2.2.2.2.3.5	1000	2. 2. 2. 5. 5. 5	1147	31. 37
961	31. 31	1001	7. 11. 13	1151	—
962	2. 13. 37	1003	17. 59	1153	—
963	3. 3. 107	1007	19. 53	1157	13. 89
964	2. 2. 241	1009	—	1159	19. 61
965	5. 193	1013	—	1163	—
966	2. 3. 7. 23	1019	—	1169	7. 167
967	—	1021	—	1171	—
968	2. 2. 2. 11. 11	1027	13. 79	1177	11. 107
969	3. 17. 19	1031	—	1181	—
970	2. 5. 97	1033	—	1183	7. 13. 13

No.	Factors.	No.	Factors.	No.	Factors.
1187	—	1339	13. 103	1489	—
1189	29. 41	1343	17. 79	1493	—
1193	—	1349	19. 71	1499	—
1199	11. 109	1351	7. 193	1501	19. 79
1201	—	1357	33. 59	1507	11. 137
1207	17. 71	1361	—	1511	—
1211	7. 173	1363	29. 47	1513	17. 89
1213	—	1367	—	1517	37. 41
1217	—	1369	37. 37	1519	7. 7. 31
1219	23. 53	1373	—	1523	—
1223	—	1379	7. 197	1529	11. 139
1229	—	1381	—	1531	—
1231	—	1387	19. 73	1537	29. 53
1237	—	1391	13. 107	1541	23. 67
1241	17. 73	1393	7. 199	1543	—
1243	11. 113	1397	11. 127	1547	7. 13. 17
1247	29. 43	1399	—	1549	—
1249	—	1403	23. 61	1553	—
1253	7. 179	1409	—	1559	—
1259	—	1411	17. 83	1561	7. 223
1261	13. 97	1417	13. 109	1567	—
1267	7. 181	1421	7. 7. 29	1571	—
1271	41. 31	1423	—	1573	11. 11. 13
1273	19. 67	1427	—	1577	19. 83
1277	—	1429	—	1579	—
1279	—	1433	—	1583	—
1283	—	1439	—	1589	7. 227
1291	—	1441	11. 131	1591	37. 43
1297	—	1447	—	1597	—
1301	—	1451	—	1601	—
1303	—	1453	—	1603	7. 229
1307	—	1457	31. 47	1607	—
1309	7. 11. 17	1459	—	1609	—
1313	13. 101	1463	7. 11. 19	1613	—
1319	—	1469	13. 113	1619	—
1321	—	1471	—	1621	—
1327	—	1477	7. 211	1627	—
1331	11. 11. 11	1481	—	1631	7. 233
1333	31. 43	1483	—	1633	23. 71
1337	7. 191	1487	—	1637	—

No.	Factors.	No.	Factors.	No.	Factors.
1639	11. 149	1789	—	1937	13. 149
1543	31. 53	1793	11. 163	1939	7. 277
1549	17. 97	1799	7. 257	1943	29. 67
1651	13. 127	1801	—	1949	—
1657	—	1807	13. 139	1951	—
1661	11. 151	1811	—	1957	19. 103
1663	—	1813	7. 7. 37	1961	37. 53
1667	—	1817	23. 79.	1963	13. 151
1669	—	1819	17. 107	1967	7. 281
1673	7. 239	1823	—	1969	11. 179
1679	23. 73	1829	31. 59	1973	—
1681	41. 41	1831	—	1979	—
1687	7. 241	1837	11. 167	1981	7. 283
1691	19. 89	1841	7. 263	1987	—
1693	—	1843	19. 97	1991	11. 181
1697	—	1847	—	1993	—
1699	—	1849	43. 43	1997	—
1703	13. 131	1853	17. 109	1999	—
1709	—	1859	11. 13. 13	2003	—
1711	29. 59	1861	—	2009	7. 7. 41
1717	17. 101	1867	—	2011	—
1721	—	1871	—	2017	—
1723	—	1873	—	2021	43. 47
1727	11. 157	1877	—	2023	7. 17. 17
1729	7. 13. 19	1879	—	2027	—
1733	—	1883	7. 269	2029	—
1739	37. 47	1889	—	2033	19. 107
1741	—	1891	31. 61	2039	—
1747	—	1893	3. 631	2041	13. 157
1751	17. 103	1897	7. 271	2047	23. 89
1753	—	1901	—	2051	7. 293
1757	7. 251	1903	11. 173	2053	—
1759	—	1907	—	2057	11. 11. 17
1763	41. 43	1909	23. 83	2059	29. 71
1769	29. 61	1913	—	2063	—
1771	7. 11. 23	1919	19. 101	2069	—
1777	—	1921	17. 113	2071	19. 109
1781	13. 137	1927	41. 47	2077	31. 67
1783	—	1931	—	2081	—
1787	—	1933	—	2083	—

No.	Factors.	No.	Factors.	No.	Factors.
2087	—	2239	—	2389	—
2089	—	2243	—	2393	—
2093	7. 13. 23	2249	13. 173	2399	—
2099	—	2252	—	2401	7. 7. 7. 7
2101	11. 191	2257	37. 61	2407	29. 83
2107	7. 7. 43	2261	7. 17. 19	2411	—
2111	—	2263	31. 73	2413	19. 127
2113	—	2267	—	2417	—
2117	29. 73	2269	—	2419	41. 59
2119	13. 163	2273	—	2423	—
2123	11. 193	2279	43. 53	2429	7. 347
2129	—	2281	—	2431	11. 13. 17
2131	—	2287	—	2437	—
2137	—	2291	29. 79	2441	—
2141	—	2293	—	2443	7. 349
2143	—	2297	—	2447	—
2147	19. 113	2299	11. 11. 19	2449	31. 79
2149	7. 307	2303	7. 7. 47	2453	11. 223
2153	—	2309	—	2459	—
2159	17. 127	2311	—	2461	23. 107
2161	—	2317	7. 331	2467	—
2167	11. 197	2321	11. 211	2471	7. 353
2171	13. 167	2323	23. 101	2473	—
2173	41. 53	2327	13. 179	2477	—
2177	7. 311	2329	17. 137	2479	37. 67
2179	—	2333	—	2483	13. 191
2183	37. 59	2339	—	2489	19. 131
2189	11. 199	2341	—	2491	47. 53
2191	7. 313	2347	—	2497	11. 217
2197	13. 13. 13.	2351	—	2501	41. 61
2201	31. 71	2353	13. 181	2503	—
2203	—	2357	—	2507	23. 109
2207	—	2359	7. 337	2509	13. 193
2209	47. 47	2363	17. 139	2513	7. 359
2213	—	2369	23. 103	2519	11. 229
2219	7. 317	2371	—	2521	—
2221	—	2377	—	2527	7. 19. 19
2227	17. 131	2381	—	2531	—
233	7. 11. 29	2383	—	2533	17. 149
[illegible]37	—	2387	7. 11. 31	2537	43. 59

No.	Factors.	No.	Factors.	No.	Factors.
2539	—	2689	—	2839	17. 167
2543	—	2693	—	2843	—
2549	—	2699	—	2849	7. 11. 37
2551	—	2701	37. 73	2851	—
2557	—	2707	—	2857	—
2561	13. 197	2711	—	2861	—
2563	11. 223	2713	—	2863	7. 409
2567	17. 151	2717	11. 13. 19	2867	47. 61
2569	7. 367	2719	—	2869	19. 151
2573	31. 83	2723	7. 389	2873	13. 13. 17
2579	—	2729	—	2879	—
2581	29. 89	2731	—	2881	43. 67
2587	13. 199	2737	7. 17. 23	2887	—
2591	—	2741	—	2891	7. 7. 59
2593	—	2743	13. 211	2893	11. 263
2597	7. 7. 53	2747	41. 67	2897	—
2599	23. 113	2749	—	2899	13. 223
2603	19. 137	2753	—	2903	—
2609	—	2759	31. 89	2909	—
2611	7. 373	2761	11. 251	2911	41. 71
2617	—	2767	—	2917	—
2621	—	2771	17. 163	2921	23. 127
2623	43. 61	2773	47. 59	2923	37. 79
2627	37. 71	2777	—	2927	—
2629	11. 239	2779	7. 397	2929	29. 101
2633	—	2783	11. 11. 23	2933	7. 419
2639	7. 13. 29	2789	—	2939	—
2641	19. 139	2791	—	2941	17. 173
2647	—	2797	—	2947	7. 421
2651	11. 241	2801	—	2951	13. 227
2653	7. 379	2803	—	2953	—
2657	—	2807	7. 401	2957	—
2659	—	2809	53. 53	2959	11. 269
2663	—	2813	29. 79	2963	—
2669	17. 157	2819	—	2969	—
2671	—	2821	7. 13. 31	2971	—
2677	—	2827	11. 257	2977	13. 229
2681	7. 383	2831	19. 149	2981	11. 271
2683	—	2833	—	2983	19. 157
2687	—	2837	—	2987	29. 103

No.	Factors.	No.	Factors.	No.	Factors.
2989	7. 7. 61	3149	47. 67	3299	—
2993	41. 73	3151	23. 137	3301	—
2999	—	3157	7. 11. 41	3307	—
3007	31. 97	3161	29. 109	3311	7. 11. 43
3011	—	3163	—	3313	—
3013	23. 131	3167	—	3317	31. 107
3017	7. 431	3169	—	3319	—
3019	—	3173	19. 167	3323	—
3023	—	3179	11. 17. 17	3329	—
3029	13. 233	3181	—	3331	—
3031	7. 433	3187	—	3337	47. 71
3037	—	3191	—	3341	13. 257
3041	—	3193	31. 103	3343	—
3043	17. 179	3197	23. 139	3347	—
3047	11. 277	3199	7. 457	3349	17. 197
3049	—	3203	—	3353	7. 479
3053	43. 71	3209	—	3359	—
3059	7. 19. 23	3211	13. 13. 19	3361	—
3061	—	3217	—	3367	7. 13. 37
3071	37. 83	3221	—	3371	—
3073	7. 439	3223	11. 293	3373	—
3077	17. 181	3227	7. 461	3377	11. 307
3079	—	3229	—	3379	31. 109
3083	—	3233	53. 61	3383	17. 199
3089	—	3239	41. 79	3389	—
3091	11. 281	3241	7. 463	3391	—
3097	19. 163	3247	17. 191	3397	43. 79
3101	7. 443	3251	—	3401	19. 179
3103	29. 107	3253	—	3403	41. 83
3107	13. 239	3257	—	3407	—
3109	—	3259	—	3409	7. 487
3113	11. 283	3263	13. 251	3413	—
3119	—	3269	7. 467	3419	13. 263
3121	—	3271	—	3421	11. 311
3127	53. 59	3277	29. 113	3427	23. 149
3131	31. 101	3281	17. 193	3431	47. 73
3133	13. 241	3283	7. 7. 67	3433	—
3137	—	3287	19. 173	3437	7. 491
3139	43. 73	3289	11. 13. 23	3439	19. 181
3143	7. 449	3293	37. 89	3443	11. 313

No.	Factors.	No.	Factors.	No.	Factors.
3449	—	3601	13. 277	3757	13. 17. 17
3451	7. 17. 29	3607	—	3761	—
3457	—	3611	23. 157	3763	53. 71
3461	—	3613	—	3767	—
3463	—	3617	—	3769	—
3467	—	3619	7. 11. 47	3773	7. 7. 7. 11
3469	—	3623	—	3779	—
3473	23. 151	3629	19. 191	3781	19. 199
3479	7. 7. 71	3631	—	3787	7. 541
3481	59. 59	3637	—	3791	17. 223
3487	11. 317	3641	11. 331	3793	—
3493	7. 499	3643	—	3797	—
3497	13. 269	3647	7. 521	3799	29. 131
3499	—	3649	41. 89	3803	—
3503	31. 113	3653	13. 281	3809	13. 293
3509	11. 11. 29	3659	—	3811	37. 103
3511	—	3667	19. 193	3817	11. 347
3517	—	3671	—	3821	—
3521	7. 503	3673	—	3823	—
3523	13. 271	3677	—	3827	43. 89
3527	—	3679	13. 283	3829	7. 547
3529	—	3683	29. 127	3833	—
3533	—	3689	7. 17. 31	3839	11. 349
3539	—	3691	—	3841	23. 167
3541	—	3697	—	3847	—
3547	—	3701	—	3851	—
3551	53. 67	3703	7. 23. 23	3853	—
3553	11. 17. 19	3707	11. 337	3857	7. 19. 29
3557	—	3709	—	3859	17. 227
3559	—	3713	47. 79	3863	—
3563	7. 509	3719	—	3869	53. 73
3569	43. 83	3721	61. 61	3871	7. 7. 79
3571	—	3727	—	3877	—
3577	7. 7. 73	3731	7. 13. 41	3881	—
3581	—	3733	—	3883	11. 353
3583	—	3737	37. 101	3887	13. 13. 23
3587	17. 211	3739	—	3889	—
3589	37. 97	3743	19. 197	3893	17. 229
3593	—	3747	3. 1249	3899	7. 557
3599	59. 61	3751	11. 11. 31	3901	47. 83

No.	Factors.	No.	Factors.	No.	Factors.
3907	—	4057	—	4207	7. 601
3911	—	4061	31. 131	4211	—
3913	7. 13. 43	4063	17. 239	4213	11. 383
3917	—	4067	7. 7. 83	4217	—
3919	—	4069	13. 313	4219	—
3923	—	4073	—	4223	41. 103
3929	—	4079	—	4229	—
3931	—	4081	7. 11. 53	4231	—
3937	31. 127	4087	61. 67	4237	19. 223
3941	7. 563	4091	—	4241	—
3943	—	4093	—	4243	—
3947	—	4097	17. 241	4247	31. 137
3949	11. 359	4099	—	4249	7. 607
3953	59. 67	4103	11. 373	4253	—
3959	37. 107	4109	7. 587	4259	—
3961	17. 233	4111	—	4261	—
3967	—	4117	23. 179	4267	17. 251
3971	11. 19. 19	4121	13. 317	4271	—
3973	29. 137	4123	7. 19. 31	4273	—
3977	41. 97	4127	—	4277	7. 13. 47
3979	23. 173	4129	—	4279	11. 389
3983	7. 569	4133	—	4283	—
3989	—	4139	—	4289	—
3991	13. 307	4141	41. 101	4291	7. 613
3997	7. 571	4147	11. 13. 29	4297	—
4001	—	4151	7. 593	4301	11. 17. 23
4003	—	4153	—	4303	13. 331
4007	—	4157	—	4307	59. 73
4009	19. 211	4159	—	4309	31. 139
4013	—	4163	23. 181	4313	19. 227
4019	—	4169	11. 379	4319	7. 617
4021	—	4171	43. 97	4321	29. 149
4027	—	4177	—	4327	—
4031	29. 139	4181	37. 113	4331	61. 71
4033	37. 109	4183	47. 89	4333	7. 619
4037	11. 367	4187	53. 79	4337	—
4039	7. 577	4189	59. 71	4339	—
4043	13. 311	4193	7. 599	4343	43. 101
4049	—	4199	13. 17. 19	4349	—
4051	—	4201	—	4351	19. 229

No.	Factors.	No.	Factors.	No.	Factors.
4357	—	4507	—	4657	—
4361	7. 7. 89	4511	13. 347	4661	59. 79
4363	—	4513	—	4663	—
4367	11. 397	4517	—	4667	13. 359
4369	17. 257	4519	—	4669	7. 23. 29
4373	—	4523	—	4673	—
4379	29. 151	4529	7. 647	4679	—
4381	13. 337	4531	23. 197	4681	31. 151
4387	41. 107	4537	13. 349	4687	43. 109
4391	—	4541	19. 239	4691	—
4393	23. 191	4543	7. 11. 59	4693	13. 19. 19
4397	—	4547	—	4697	7. 11. 61
4399	53. 83	4549	—	4699	37. 127
4403	7. 17. 37	4553	29. 157	4703	—
4409	—	4559	57. 97	4707	—
4411	11. 401	4561	—	4709	17. 227
4421	—	4567	—	4711	7. 673
4423	—	4571	7. 653	4717	53. 89
4427	19. 233	4573	17. 269	4721	—
4429	43. 103	4577	23. 199	4723	—
4433	11. 13. 31	4579	19. 241	4727	29. 163
4439	23. 193	4583	—	4729	—
4441	—	4589	13. 353	4733	—
4447	—	4591	—	4739	7. 677
4451	—	4597	—	4741	11. 431
4453	61. 73	4601	43. 107	4747	47. 101
4457	—	4603	—	4751	—
4459	7. 7. 7. 13	4607	17. 271	4753	7. 7. 97
4463	—	4609	11. 419	4757	67. 71
4469	41. 109	4613	7. 659	4759	—
4471	17. 263	4619	31. 149	4763	11. 433
4477	11. 11. 37	4621	—	4769	19. 251
4481	—	4627	7. 661	4771	13. 367
4483	—	4631	11. 421	4777	17. 281
4487	7. 641	4633	41. 113	4781	7. 683
4489	67. 67	4637	—	4783	—
4493	—	4639	—	4787	—
4499	11. 409	4643	—	4789	—
4501	7. 643.	4649	—	4793	—
4503	—	4651	—	4799	—

No.	Factors.	No.	Factors.	No.	Factors.
4801	—	4957	—	5107	—
4807	11. 19. 23	4961	11. 11. 41	5111	19. 269
4811	17. 283	4963	7. 709	5113	—
4813	—	4967	—	5117	7. 17. 43
4817	—	4969	—	5119	—
4819	61. 79	4973	—	5123	47. 109
4823	7. 13. 53	4979	13. 383	5129	23. 223
4829	11. 439	4981	17. 293	5131	7. 733
4831	—	4987	—	5137	11. 467
4837	7. 691	4991	7. 23. 31	5143	37. 139
4841	47. 103	4993	—	5147	—
4843	29. 167	4997	19. 263	5149	19. 271
4847	37. 131	4999	—	5153	—
4853	23. 211	5003	—	5159	7. 11. 67
4859	43. 113	5009	—	5161	13. 397
4861	—	5011	—	5167	—
4867	31. 157	5017	29. 173	5171	—
4871	—	5021	—	5173	7. 739
4873	11. 443	5023	—	5177	31. 167
4877	—	5027	11. 457	5179	—
4879	7. 17. 41	5029	47. 107	5183	71. 73
4883	19. 257	5033	7. 719	5189	—
4889	—	5039	—	5191	29. 179
4891	67. 73	5041	71. 71	5197	—
4897	59. 83	5047	7. 7. 103	5201	7. 743
4901	13. 13. 29	5051	—	5203	11. 11. 43
4903	—	5053	31. 193	5207	41. 127
4907	7. 701	5057	13. 389	5209	—
4909	—	5059	—	5213	13. 401
4913	17. 17. 17	5063	61. 83	5219	17. 307
4919	—	5069	37. 137	5221	23. 227
4921	7. 19. 37	5071	11. 461	5227	—
4927	13. 379	5077	—	5231	—
4931	—	5081	—	5233	—
4933	—	5083	13. 17. 23	5237	—
4937	—	5087	—	5239	13. 13. 31
4939	11. 449	5089	7. 727	5243	7. 7. 107
4943	—	5093	11. 463	5249	29. 181
4949	7. 7. 101	5099	—	5251	59. 89
951	—	5101	—	5257	7. 751

No.	Factors.	No.	Factors.	No.	Factors.
5261	—	5411	7. 773	5561	67. 83
5263	19. 227	5413	—	5563	—
5267	23. 229	5417	—	5567	19. 293
5269	11. 479	5419	—	5569	—
5273	—	5423	11. 17. 29	5573	—
5279	—	5429	61. 89	5579	7. 797
5281	—	5431	—	5581	—
5287	17. 311	5437	—	5587	37. 151
5291	11. 13. 37	5441	—	5591	—
5293	69. 79	5443	—	5593	7. 17. 47
5297	—	5447	13. 419	5597	29. 193
5299	7. 757	5449	—	5599	11. 509
5303	—	5453	7. 19. 41	5603	13. 431
5309	—	5459	53. 103	5609	71. 79
5311	47. 113	5461	43. 127	5611	31. 181
5317	13. 409	5467	7. 11. 71	5617	41. 137
5321	17. 313	5471	—	5621	7. 11. 73
5323	—	5473	13. 421	5623	—
5327	7. 761	5477	—	5627	17. 331
5329	73. 73	5479	—	5629	13. 433
5333	—	5483	—	5633	43. 131
5339	19. 281	5489	11. 499	5639	—
5341	7. 7. 109	5491	17. 17. 19	5641	—
5347	—	5497	23. 239	5647	—
5351	—	5501	—	5651	—
5353	53. 101	5503	—	5653	—
5357	11. 487	5507	—	5657	—
5359	23. 233	5509	7. 787	5659	—
5363	31. 173	5513	37. 149	5663	7. 809
5369	7. 13. 59	5519	—	5669	—
5371	41. 131	5521	—	5671	53. 107
5377	19. 283	5527	—	5681	13. 19. 23
5381	—	5531	—	5683	—
5383	7. 769	5533	11. 503	5687	11. 11. 47
5387	—	5537	7. 7. 113	5689	—
5389	17. 317	5539	29. 191	5693	—
5393	—	5543	23. 241	5699	41. 139
5399	—	5549	31. 179	5701	—
5401	11. 491	5551	7. 13. 61	5707	13. 439
5407	—	5557	—	5711	—

No.	Factors.	No.	Factors.	No.	Factors.
5713	29. 197	5861	—	6011	—
5717	—	5863	11. 13. 41	6013	7. 859
5719	7. 19. 43	5867	—	6017	11. 547
5723	59. 97	5869	—	6019	13. 463
5729	17. 137	5873	7. 839	6023	19. 317
5731	11. 521	5879	—	6029	—
5737	—	5881	—	6031	37. 163
5741	—	5887	7. 29. 29	6037	—
5743	—	5891	43. 137	6041	7. 863
5747	7. 821	5893	71. 83	6043	—
5749	—	5897	—	6047	—
5753	11. 523	5899	17. 347	6049	23. 263
5759	13. 443	5903	—	6053	—
5761	7. 823	5909	19. 311	6059	73. 83
5763	3. 17. 113	5911	23. 257	6061	11. 19. 29
5767	73. 79	5917	61. 97	6067	—
5771	29. 199	5921	31. 191	6071	13. 467
5773	23. 251	5923	—	6073	—
5777	53. 109	5927	—	6077	59. 103
5779	—	5929	7. 7. 11. 11	6079	—
5783	—	5933	17. 349	6083	7. 11. 79
5789	7. 827	5939	—	6089	—
5791	—	5941	13. 457	6091	—
5797	11. 17. 31	5947	19. 313	6097	7. 13. 67
5801	—	5951	11. 541	6001	—
5803	7. 829	5953	—	6103	17. 359
5807	—	5957	7. 23. 37	6107	31. 197
5809	37. 157	5959	59. 101	6109	41. 149
5813	—	5963	67. 89	6113	—
5819	11. 23. 23	5969	47. 127	6119	29. 211
5821	—	5971	7. 853	6121	—
5827	—	5977	43. 139	6127	11. 557
5831	7. 7. 7. 17	5981	—	6131	—
5833	19. 307	5983	31. 193	6133	—
5837	13. 449	5987	—	6137	17. 19. 19
5839	—	5989	53. 113	6139	7. 877
5843	—	5993	13. 461	6143	—
5849	—	5999	7. 857	6149	11. 13. 43
851	—	6001	17. 353	6151	—
857	—	6007	—	6157	47. 131

No.	Factors.	No.	Factors.	No.	Factors.
6161	61. 101	6311	—	6463	23. 281
6163	—	6313	59. 107	6467	29. 223
6167	7. 881	6317	—	6469	—
6169	31. 199	6319	71. 89	6473	—
6173	—	6323	—	6479	11. 19. 31
6179	37. 167	6329	—	6481	—
6181	7. 883	6331	13. 487	6487	13. 499
6187	27. 269	6337	—	6491	—
6191	41. 151	6341	17. 373	6493	43. 151
6193	11. 563	6343	—	6499	67. 97
6197	—	6347	11. 577	6503	7. 929
6199	—	6349	7. 907	6509	23. 283
6203	—	6353	—	6511	17. 383
6209	7. 887	6359	—	6517	7. 7. 7. 19
6211	—	6361	—	6521	—
6217	—	6367	—	6523	11. 593
6221	—	6371	23. 277	6527	61. 107
6223	7. 7. 127	6373	—	6529	—
6227	13. 479	6377	7. 911	6533	47. 139
6229	—	6379	—	6539	13. 503
6233	23. 271	6383	13. 491	6541	31. 211
6239	17. 367	6389	—	6547	—
6241	79. 79	6391	7. 11. 83	6551	—
6247	—	6401	37. 173	6553	—
6251	7. 19. 47	6403	19. 337	6557	79. 83
6253	13. 13. 37	6407	43. 149	6559	7. 937
6257	—	6409	13. 17. 29	6563	—
6259	11. 569	6413	11. 11. 53	6569	—
6263	—	6419	7. 7. 131	6571	—
6269	—	6421	—	6577	—
6271	—	6427	—	6581	—
6277	—	6431	59. 109	6583	29. 227
6281	11. 571	6433	7. 919	6587	7. 941
6283	61. 103	6437	41. 157	6589	11. 599
6287	—	6439	47. 137	6593	19. 347
6289	19. 331	6443	17. 379	6599	—
6293	7. 29. 31	6449	—	6601	7. 23. 41
6299	—	6451	—	6607	—
6301	—	6457	11. 587	6611	11. 601
6307	7. 17. 53	6461	7. 13. 71	6613	17. 389

No.	Factors.	No.	Factors.	No.	Factors.
6617	13. 509	6767	67. 101	6917	—
6619	—	6769	7. 967	6919	11. 17. 37
6623	37. 179	6773	13. 521	6923	7. 23. 43
6629	7. 947	6779	—	6929	13. 13. 41
6631	19. 349	6781	—	6931	29. 239
6637	—	6787	11. 617	6937	7. 991
6641	29. 229	6791	—	6941	11. 631
6643	7. 13. 73	6793	—	6943	53. 131
6647	17. 17. 23	6797	7. 971	6947	—
6649	61. 109	6799	13. 523	6949	—
6653	—	6803	—	6953	17. 409
6659	—	6809	11. 619	6959	—
6661	—	6811	7. 7. 139	6961	—
6667	59. 113	6817	17. 401	6967	—
6671	7. 953	6821	19. 359	6971	—
6673	—	6823	—	6973	19. 367
6677	11. 607	6827	—	6977	—
6679	—	6829	—	6979	7. 997
6683	41. 163	6833	—	6983	—
6689	—	6839	7. 977	6989	29. 241
6691	—	6841	—	6991	—
6697	37. 181	6847	41. 167	6997	—
6701	—	6851	13. 17. 31	7001	—
6703	—	6853	7. 11. 89	7003	47. 149
6707	19. 353	6857	—	7007	7. 7. 11. 13
6709	—	6859	19. 19. 19	7009	43. 163
6713	7. 7. 137	6863	—	7013	—
6719	—	6869	—	7019	—
6721	11. 13. 47	6871	—	7021	7. 17. 59
6727	7. 31. 31	6877	13. 23. 23	7027	—
6731	53. 127	6881	7. 983	7031	79. 89
6733	—	6883	—	7033	13. 541
6737	—	6887	71. 97	7037	31. 227
6739	23. 293	6889	83. 83	7039	—
6743	11. 613	6893	61. 113	7043	—
6749	17. 397	6899	—	7049	7. 19. 53
6751	43. 157	6901	67. 103	7051	11. 641
6757	29. 233	6907	—	7057	—
6761	—	6911	—	7061	23. 307
6763	—	6913	31. 223	7063	7. 1009

No.	Factors.	No.	Factors.	No.	Factors.
7067	37. 191	7217	7. 1031	7367	53. 139
7069	—	7219	—	7369	—
7073	11. 643	7223	31. 233	7373	73. 101
7079	—	7229	—	7379	47. 157
7081	73. 97	7231	7. 1033	7381	11. 11. 61
7087	19. 373	7237	—	7387	83. 89
7091	7. 1013	7241	13. 557	7391	19. 389
7093	41. 173	7243	—	7393	—
7097	47. 151	7247	—	7397	13. 569
7099	31. 229	7249	11. 659	7399	7. 7. 151
7103	—	7253	—	7403	11. 673
7109	—	7259	7. 17. 61	7409	31. 239
7111	13. 547	7261	53. 137	7411	—
7117	11. 647	7267	13. 13. 43	7417	—
7121	—	7271	11. 661	7421	41. 181
7123	17. 419	7273	7. 1039	7423	13. 571
7127	—	7277	19. 383	7427	7. 1061
7129	—	7279	29. 251	7429	17. 19. 23
7133	7. 1019	7283	—	7433	—
7139	11. 11. 59	7289	37. 197	7439	43. 173
7141	37. 193	7291	23. 317	7441	7. 1063
7147	7. 1021	7297	—	7447	11. 677
7151	—	7301	7. 7. 149	7451	—
7153	23. 311	7303	67. 109	7453	29. 257
7157	17. 421	7307	—	7457	—
7159	—	7309	—	7459	—
7163	13. 19. 29	7313	71. 103	7463	17. 439
7169	67. 107	7319	13. 563	7469	7. 11. 97
7171	71. 101	7321	—	7471	31. 241
7177	—	7327	17. 431	7477	—
7181	43. 167	7331	—	7481	—
7183	11. 653	7333	—	7483	7. 1069
7187	—	7337	11. 23. 29	7487	—
7189	7. 13. 79	7339	41. 179	7489	—
7193	—	7343	7. 1049	7493	59. 127
7199	23. 313	7349	—	7499	—
7201	19. 379	7351	—	7501	13. 577
7207	—	7357	7. 1051	7507	—
7211	—	7361	17. 433	7511	7. 29. 37
7213	—	7363	37. 199	7513	11. 683

No.	Factors.	No.	Factors.	No.	Factors.
7517	—	7667	11. 17. 41	7817	—
7519	73. 103	7669	—	7819	7. 1117
7523	—	7673	—	7823	—
7529	—	7679	7. 1097	7829	—
7531	17. 443	7681	—	7831	41. 191
7537	—	7687	—	7837	17. 461
7541	—	7691	—	7841	—
7543	19. 397	7693	7. 7. 157	7843	11. 23. 31
7547	—	7697	43. 179	7847	7. 19. 59
7549	—	7699	—	7849	47. 167
7553	7. 13. 83	7703	—	7853	—
7559	—	7709	13. 593	7859	29. 271
7561	—	7711	11. 701	7861	7. 1123
7567	7. 23. 47	7717	—	7867	—
7571	67. 113	7721	7. 1103	7871	17. 463
7573	—	7723	—	7873	—
7577	—	7727	—	7877	—
7579	11. 13. 53	7729	59. 131	7879	—
7583	—	7733	11. 19. 37	7883	—
7589	—	7739	71. 109	7889	7. 7. 7. 23
7591	—	7741	—	7891	13. 607
7597	71. 107	7747	61. 127	7897	53. 149
7601	11. 691	7751	23. 337	7901	—
7603	—	7753	—	7903	7. 1129
7607	—	7757	—	7907	—
7609	7. 1087	7759	—	7909	11. 719
7613	23. 331	7763	7. 1109	7913	41. 193
7619	19. 401	7769	17. 457	7919	—
7621	—	7771	19. 409	7921	89. 89
7627	29. 263	7777	7. 11. 101	7927	—
7631	13. 587	7781	31. 251	7931	7. 11. 103
7633	17. 449	7783	43. 181	7933	—
7637	7. 1091	7787	13. 599	7937	—
7639	—	7789	—	7939	17. 467
7643	—	7793	—	7943	13. 13. 47
7649	—	7799	11. 709	7949	—
7651	7. 1093	7801	29. 269	7951	—
7657	13. 19. 31	7807	37. 211	7957	73. 109
7661	47. 163	7811	73. 107	7961	19. 419
7663	79. 97	7813	13. 601	7963	—

No.	Factors.	No.	Factors.	No.	Factors.
7967	31. 257	8117	—	8267	7. 1181
7969	13. 613	8119	23. 353	8269	—
7973	7. 17. 67	8123	—	8273	—
7979	79. 101	8129	11. 739	8279	17. 487
7981	23. 347	8131	47. 173	8281	7. 7. 13. 13
7987	7. 7. 163	8137	79. 103	8287	—
7991	61. 131	8141	7. 1163	8291	—
7993	—	8143	17. 479	8293	—
7997	11. 727	8147	—	8297	—
7999	19. 421	8149	29. 281	8299	43. 193
8003	53. 151	8153	31. 263	8303	19. 19. 23
8009	—	8159	41. 199	8309	7. 1187
8011	—	8161	—	8311	—
8017	—	8167	—	8317	—
8021	13. 617	8171	—	8321	53. 157
8023	71. 113	8173	11. 743	8323	7. 29. 41
8027	23. 349	8177	13. 17. 37	8327	11. 757
8029	7. 31. 37	8179	—	8329	—
8033	29. 277	8183	7. 7. 167	8333	13. 641
8039	—	8189	19. 431	8339	31. 269
8041	11. 17. 43	8191	—	8341	19. 439
8047	13. 619	8197	7. 1171	8347	17. 491
8051	83. 97	8201	59. 139	8351	7. 1193
8053	—	8203	13. 631	8353	—
8057	7. 1151	8207	29. 283	8357	61. 137
8059	—	8209	—	8359	13. 643
8063	11. 733	8213	43. 191	8363	—
8069	—	8219	—	8369	—
8071	7. 1153	8221	—	8371	11. 761
8077	41. 197	8227	19. 433	8377	—
8081	—	8231	—	8381	17. 17. 29
8083	59. 137	8233	—	8383	83. 101
8087	—	8237	—	8387	—
8089	—	8239	7. 11. 107	8389	—
8093	—	8243	—	8393	7. 11. 109
8099	7. 13. 89	8249	73. 113	8399	37. 227
8101	—	8251	37. 223	8401	31. 271
8107	11. 11. 67	8257	23. 359	8407	7. 1201
8111	—	8261	11. 751	8411	13. 647
8113	7. 19. 61	8263	—	8413	47. 179

No.	Factors.	No.	Factors.	No.	Factors.
8417	19. 443	8567	13. 659	8719	—
8419	—	8569	11. 19. 41	8723	11. 13. 61
8423	—	8573	—	8729	7. 29. 43
8429	—	8579	23. 373	8731	—
8431	—	8581	—	8737	—
8437	11. 13. 59	8587	31. 277	8741	—
8441	23. 367	8591	11. 11. 71	8743	7. 1249
8443	—	8593	13. 661	8747	—
8447	—	8597	—	8749	13. 673
8449	7. 17. 71	8599	—	8753	—
8453	79. 107	8603	7. 1229	8759	19. 461
8459	11. 769	8609	—	8761	—
8461	—	8611	79. 109	8767	11. 797
8467	—	8617	7. 1231	8771	7. 7. 179
8471	43. 197	8621	37. 233	8773	31. 283
8473	37. 229	8623	—	8777	67. 131
8477	7. 7. 173	8627	—	8779	—
8479	61. 139	8629	—	8783	—
8483	17. 499	8633	97. 89	8789	11. 17. 47
8489	13. 653	8639	53. 163	8791	59. 149
8491	7. 1213	8641	—	8797	19. 463
8497	29. 293	8647	—	8801	13. 677
8501	—	8651	41. 211	8803	—
8503	11. 773	8653	17. 509	8807	—
8507	47. 181	8657	11. 787	8809	23. 383
8509	67. 127	8659	7. 1237	8813	7. 1259
8513	—	8663	—	8819	—
8519	7. 1217	8669	—	8821	—
8521	—	8671	13. 23. 29	8827	7. 13. 97
8527	—	8677	—	8831	—
8531	19. 449	8681	—	8833	11. 11. 73
8533	7. 23. 53	8683	19. 457	8837	—
8537	—	8687	7. 17. 73	8839	—
8539	—	8689	—	8843	37. 239
8543	—	8693	—	8849	—
8549	83. 103	8699	—	8851	53. 167
8551	17. 503	8701	7. 11. 113	8857	17. 521
8557	43. 199	8707	—	8861	—
8561	7. 19. 67	8711	31. 281	8863	—
568	—	8713	—	8867	—

No.	Factors.	No.	Factors.	No.	Factors.
8869	7. 7. 181	9019	29. 311	9169	53. 173
8873	19. 467	9023	7. 1289	9173	—
8879	13. 683	9029	—	9179	67. 137
8881	83. 107	9031	11. 821	9181	—
8887	—	9037	7. 1291	9187	—
8891	17. 523	9041	—	9191	91. 101
8893	—	9043	—	9193	29. 317
8897	7. 31. 41	9047	83. 109	9197	17. 541
8899	11. 809	9049	—	9199	—
8903	29. 307	9053	11. 823	9203	—
8909	59. 151	9059	—	9209	—
8911	7. 19. 67	9061	13. 17. 41	9211	61. 151
8917	37. 241	9067	—	9217	13. 709
8921	11. 811	9071	47. 193	9221	—
8923	—	9073	43. 211	9223	23. 401
8927	79. 113	9077	29. 313	9227	—
8929	—	9079	7. 1297	9229	11. 839
8933	—	9083	31. 293	9233	7. 1319
8939	7. 1277	9089	61. 149	9239	—
8941	—	9091	—	9241	—
8947	23. 389	9097	11. 827	9247	7. 1321
8951	—	9101	19. 479	9251	11. 29. 29
8953	7. 1279	9103	—	9253	19. 487
8957	13. 13. 53	9107	7. 1301	9257	—
8959	17. 17. 31	9109	—	9259	47. 197
8963	—	9113	13. 701	9263	59. 157
8969	—	9119	11. 829	9269	13. 23. 31
8971	—	9121	7. 1303	9271	73. 127
8977	47. 191	9127	—	9277	—
8981	7. 1283	9131	23. 397	9281	—
8983	13. 691	9133	—	9283	—
8987	11. 19. 43	9137	—	9287	37. 251
8989	89. 101	9139	13. 19. 37	9289	7. 1327
8993	17. 23. 23	9143	41. 233	9293	—
8999	—	9149	7. 1307	9299	17. 547
9001	—	9151	—	9301	71. 131
9007	—	9157	—	9307	41. 217
9011	—	9161	—	9311	—
9013	—	9163	7. 7. 11. 17	9313	67. 139
9017	71. 127	9167	89. 103	9317	7. 11. 11.

No.	Factors.	No.	Factors.	No.	Factors.
9319	—	9473	—	9623	—
9323	—	9479	—	9629	—
9329	19. 491	9481	19. 499	9631	—
9331	7. 31. 43	9487	53. 179	9637	23. 419
9337	—	9491	—	9641	31. 311
9341	—	9493	11. 863	9643	—
9343	—	9497	—	9647	11. 877
9347	13. 719	9499	7. 23. 59	9649	—
9349	—	9503	13. 17. 43	9653	7. 7. 197
9353	47. 199	9509	37. 257	9659	13. 743
9359	7. 7. 191	9511	—	9661	—
9361	11. 23. 37	9517	31. 307	9667	7. 1381
9367	17. 19. 29	9521	—	9671	19. 509
9371	—	9523	89. 107	9673	17. 569
9373	7. 13. 103	9527	7, 1361	9677	—
9377	—	9529	13. 733	9679	—
9379	83. 113	9533	—	9683	23. 421
9383	11. 853	9539	—	9689	—
9389	41. 229	9541	7. 29. 47	9691	11. 881
9391	—	9547	—	9697	—
9397	—	9551	—	9701	89. 109
9401	7. 17. 79	9553	41. 233	9703	31. 313
9403	—	9557	19. 503	9707	17. 571
9407	23. 409	9559	11. 11. 79	9709	7. 19. 73
9409	97. 97	9563	73. 131	9713	11. 883
9413	—	9569	7. 1367	9719	—
9419	—	9571	17. 563	9721	—
9421	—	9577	61. 157	9727	71. 137
9427	11. 857	9581	11. 13. 67	9731	37. 263
9431	—	9583	7. 37. 37	9733	—
9433	—	9587	—	9737	7. 13. 107
9437	—	9589	43. 223	9739	—
9439	—	9593	53. 181	9743	—
9443	7. 19. 71	9599	29. 331	9749	—
9449	11. 859	9601	—	9751	7. 7. 109
9451	13. 727	9607	13. 739	9757	11. 887
9457	7. 7. 193	9611	7. 1373	9761	43. 227
9461	—	9613	—	9763	13. 751
9467	—	9617	59. 163	9767	—
9469	17. 557	9619	—	9769	—

No.	Factors.	No.	Factors.	No.	Factors.
9773	29. 337	9851	—	9923	—
9779	7. 11. 127	9853	59. 167	9931	—
9781	—	9857	—	9937	19. 523
9787	—	9859	—	9941	—
9791	—	9863	7. 1409	9943	61. 163
9793	7. 1399	9869	71. 139	9947	7. 7. 7. 29
9797	97. 101	9871	—	9949	—
9799	41. 239	9877	7. 17. 83	9953	37. 269
9803	—	9881	41. 241	9959	23. 433
9809	17. 577	9883	—	9961	7. 1423
9811	—	9887	—	9967	—
9817	—	9889	11. 29. 31	9971	13. 13. 59
9821	7. 23. 61	9893	13. 761	9973	—
9823	11. 19. 47	9899	19. 521	9977	11. 907
9827	31. 317	9901	—	9979	17. 587
9829	—	9907	—	9983	67. 149
9833	—	9911	11. 17. 53	9989	7. 1427
9839	—	9913	23. 431	9991	97. 103
9841	13. 757	9917	47. 211	9997	13. 769
9847	43. 229	9919	7. 13. 109		

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

By attentive mental discipline the above table may be greatly extended; the square of 125 and intermediate numbers may be rendered, with a little application, nearly as familiar as that of 12; which extension will be found very useful in the daily business of buying and selling small amounts. This plan consists in applying the principle of algebraic computation to common figures; namely, calculating from the left to the right, taking the largest amounts first. Suppose we have to calculate 123 yards at 123 cents: Commencing at the left hand we say $12 \times 12 = 144$ hund's.

Then 3×12 and $3 \times 12 =$ 72 tens.

And lastly, $3 \times 3 =$ 9 units

*Result $151.29

* In algebra we would express the above thus: $ab \times ab$ is

$$\begin{array}{c} ab \\ ab \\ \hline aa + ab + bb \\ +ab \\ \hline 2a + 2ab + 2b = x \end{array}$$

or $151.29.

Again: when the numbers differ, as 121 by 119, we may say,

$$\begin{array}{lll} 11\times 12= & & 132 \text{ hundreds.} \\ \left.\begin{array}{l} 9\times 12=108 \\ 1\times 11=\ \ 11 \end{array}\right\} & = & 119 \text{ tens.} \\ 9\times\ 1= & & 9 \text{ units.} \\ \hline & & \$14399 \end{array}$$

But if we adopt the algebraic operation still farther, and use the signs + more, and — less, the calculation will stand thus:

$$\left.\begin{array}{l} 121=120+1 \\ 119=120-1 \end{array}\right\} \begin{array}{l} \text{We then proceed } 12\times 12=144 \text{ hundreds.} \\ \left.\begin{array}{l} 1\times 12 \textit{ more} \\ 1\times 12 \textit{ less} \end{array}\right\} \text{balance each other, } \quad 0 \text{ tens.} \\ -1\times +1= 1 \text{ less, so } \textit{subtract } \ 1 \text{ unit.} \\ \hline \$14399 \text{ as above} \end{array}$$

TABLE OF EXCHANGE.

Accounts are kept in the United States in Federal money, dollars, dimes, cents and mills.

10 mills . = 1 cent,
10 cents . = 1 dime,
10 dimes . = 1 dollar.

Accounts in *England* are kept in pounds, shillings, pence and farthings, sterling money.

4 farthings . = 1 penny,
12 pence . = 1 shilling,
20 shillings . = 1 £ sterling.
If exchange is at par \$1 = 4½ shillings,
or, \$40 = 9 £ sterling.

If it is said exchange is 10 per cent. premium, it means
9 £ = 40 dollars, and 10 per cent. more, or
100 = 110
900 = 4400 or 9 £ sterling = 44 dollars.

Accounts in *Spain* are kept in dollars, rials of plate, rials vellon, quartas, ochavos, maravedies.

1 dollar = 10 rials of plate,
1 dollar = 20 rials vellon,
1 rial vellon = 8 quartas,
1 quarta = 2 ochavos,
1 ochavo = 2 maravedies.

Accounts in *Spanish America* are kept in dollars, rials of plate, medios and quartillos.

4 quartillos, = 1 medio,
2 medios = 1 rial of plate,
8 rials of plate = 1 dollar.

The Spanish and Mexican dollars are equal in value with that of the United States of America.

Accounts are kept in *France* in francs, decimes and centimes.

10 centimes = 1 decime,
10 decimes = 1 franc,
1 franc = $18\frac{3}{4}$ cents,
$533\frac{1}{3}$ centimes = \$1.

Accounts are kept in *Portugal* in millreas and reas.

1 millrea = 1000 reas,
400 reas = 1 crusado of exchange.
1 millrea = \$1.24.
400 reas = $49\frac{3}{5}$ or 1 crusado of exchange.

Accounts are kept in the *Netherlands* in florins and centimes.

1 florin = 100 centimes,
1 florin = 40 cents.

Accounts are kept in *Hamburg* in marks, schillings, pfenings and bancos.

12 pfenings .	=	1 schilling,
16 schillings .	=	1 mark banco,
3 marks banco	=	1 rix dollar,
1 mark banco	=	$33\frac{1}{3}$ cents.

Accounts are kept in *Sweden* in rix dollars specie, skillings, and rundstyckens, or ore.

12 rundstyckens	=	1 skilling,
48 skillings	=	1 rix dollar specie.

The rix dollar specie is equal in value to the dollar of America.

Exchange business of Russia, Austria, Prussia, Poland and Germany, are transacted by the above related intermediate places.

Foreign coins are not a legal tender in the payment of debts. In calculating the duties *ad valorem*, on invoices given in foreign countries, the following are the rates settled by the laws of the United States.

England,	the	Pound sterling	to be estimated	$4 44
France,	"	Franc	"	$18\frac{3}{4}$
Netherlands,	"	Florin	"	40
Hamburg,	"	Mark banco	"	$33\frac{1}{3}$
Denmark,	"	Rix dollar	"	1 00
Spain,	"	Rial of plate	"	10
do.	"	Rial vellon	"	5
Portugal,	"	Millrea	"	1 24
Ireland,	"	Pound	"	4 10
China,	"	Tale	"	1 48
India,	"	Pagoda	"	1 94
Bengal,	"	Rupee	"	$55\frac{1}{2}$

The duties on foreign importation are at present calculated 20 per cent. *ad valorem*, but they may be changed by law.

The price current of all commodities, as also the rate of exchange from one place to another, are noted in the com-

mercial transactions of each day; but it will be observed that the *fiscal unit* of each place is not expressed in these reports of the rate of exchange, as

Exchange on	England,	8½ per cent. premium.
"	France,	5 francs 28 centimes.
"	Hamburg,	34 cents.
"	the Netherlands,	39 cents.

Which means for England, \$1=4½ shill. and 8½ per cent. more, or 9£=\$40 and 8½ per cent. more.
" France, \$1=5 francs 28 centimes.
" Hamburg, 1 mark banco=34 cents.
" the Netherlands, 1 florin=39 cents.

and according to these given proportions, exchange is calculated in *practical* business transactions.

RELATIVE PROPORTION OF MONEY OF THE UNITED STATES.

Pennsylvania, N. Jersey, Del. and Maryland.	New York and N. Carolina.	Virginia and N. E. States.	S. Carolina and Georgia.
£3=8\$	£2=5\$	£3=10\$	£7=30\$
sh. 3=40 c.	sh. 2=25 c.	sh. 3=50 c.	sh. 7=150 c.
d. 9=10 c.	d. 24=25 c.	d. 18=25 c.	d. 14=25 c.
Penn'a. 15=16	New York.	N. Y. 16=15	Pennsylvania.
5= 4	Virginia.	4= 3	Virginia.
45=28	S. Carolina.	12= 7	S. Carolina.
Va. 4= 5	Pennsylvania.	S. C. 28=45	Pennsylvania.
3= 4	New York.	7=12	New York.
9= 7	S. Carolina.	7= 9	Virginia.

TROY WEIGHT.

Pound. lb.	Ounce. oz.	Pennyweight. dwt.	Grain. gr.
1	12	240	5760
	1	20	480
		1	24

By this weight gold, silver, jewels, and liquors, are weighed.

AVOIRDUPOIS WEIGHT.

Ton. T.	Hundred weight. cwt.	Quarter. qr.	Pound. lb.	Ounce. oz.	Drachm. dr.
1	20	80	2240	35840	573440
	1	4	112	1792	28672
		1	28	448	7468
			1	16	256
				1	16

By this weight are weighed things of a coarse drossy nature, that are bought and sold by weight; and all metals, but silver and gold.

APOTHECARIES WEIGHT.

Pound.	Ounce.	Drachm.	Scruple.	Grain.
1	12	96	288	5760
	1	8	24	480
		1	3	60
			1	20

By this weight apothecaries mix their medicines, but buy and sell by avordupois weight.

THINGS BOUGHT AND SOLD BY THE DOZEN, GROSS, &c.

Great gross. g. gro.	Common gross. gro.	Dozen. doz.	Particulars. prs.
1	12	144	1728
	1	12	144
		1	12

LONG MEASURE.

Circle.	Degree.	League.	Geographical miles.	Statute miles.	Furlong.	Chain.	Perch.	Yard.	Foot.	Link.	Inch.	Barley Corn.
1	360	7200	21600	25025	172800	1728000	6912000	38016000	114048000	1728000000	1368576000	4105728000
	1	20	60	69½	480	4800	19200	105600	316800	480090	3801600	11404800
	2	1		139								
			3	"	24	240	960	5280	15840	24000	190080	5700240
			1	"	8	80	320	1760	5280	8000	63360	190080
				"	1	10	40	220	660	1000	7920	28760
						1	4	22	66	100	792	2376
							1	5½	16½	25	198	594
							2	11	33	50	396	1188
								1	3	"	36	108
									1	"	12	36
										"	1	3

A hand is a measure of four inches, commonly used by horse dealers.
A fathom is a measure of six feet, by which the depths of water or wells and mines are measured.

LIQUID MEASURE.

Tun. T.	Pipes. pi. or bt.	Hogs-heads. hhd.	Gallons. gal.	Quarts. qt.	Pints. pt.
1	2	4	252	1008	2016
	1	2	126	504	1008
		1	63	252	504
			1	4	8
				1	2

CLOTH MEASURE.

Yard. yd.	Quar-ters. qr.	Nail. na.	Inches. in.		Ell.	Yard. yd.
1	4	16	36	Flemish	4	3
	1	4	9	English	4	5
		1	$2\frac{1}{4}$	Hamburg	8	5

SQUARE MEASURE.

Mile.	Acre.	Rood.	Chain	Perch.	Yard.	Foot.	Link.	Inch.	Barley Corn.
1	640	2560	6400	102400	3097600	2787400	64000000	40144890	37130406400
	1	4	10	160	4840	43560	100000	6272640	56453760
		1	"	40	1210	10890	"	1568160	14113440
			1	"	"	"	10000	"	"
				1	$30\frac{1}{4}$	$272\frac{1}{4}$	"	39204	352836
				4	121	1089	"	"	"
					1	9	"	1296	11664
						1	"	144	432
								1	9

DRY MEASURE.

Bushel. bu.	Peck. P.	Quart. qt.	Pint. pt.
1	4	32	64
	1	8	16
		1	2

MOTION, OR CIRCLE MEASURE.

Circle.	Sine.	Degree. °	Minute. ′	Second. ″
1	12	360	21600	1296000
	1	30	1800	108000
		1	60	3600
			1	60

DIVISION OF TIME.

Year.	Months.	Weeks.	Days.	Hours.	Minutes.	Seconds.
1	12	52	365	8760	525600	19536000
	1	4				
		1	7			
			1	24	1440	86400
				1	60	3600
					1	60

365 days 6 hours } 1 year nearly.
13 lunar months, 1 day, 6 hours }

THE END.